BATS: Achieving the Capacity of Networks with Packet Loss

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Outline

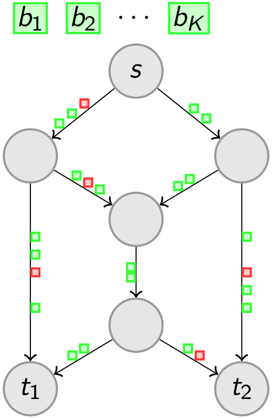
1. Problem

2. BATS Codes
   - Encoding and Decoding
   - Degree Distribution
   - Achievable Rates

3. Recent Developments

Transmission through Packet Networks (Erasure Networks)

One 20MB file ≈ 20,000 packets

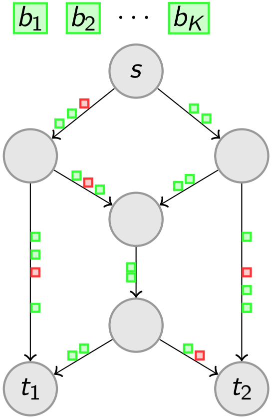


Transmission through Packet Networks (Erasure Networks)

One 20MB file ≈ 20,000 packets

A practical solution

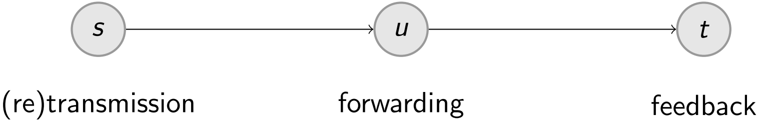
- low computational and storage costs
- high transmission rate
- small protocol overhead



Routing Networks

Retransmission

- Example: TCP
- Not scalable for multicast
- Cost of feedback



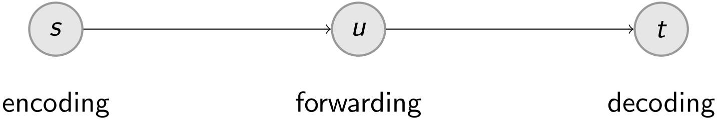
Routing Networks

Retransmission

- Example: TCP
- Not scalable for multicast
- Cost of feedback

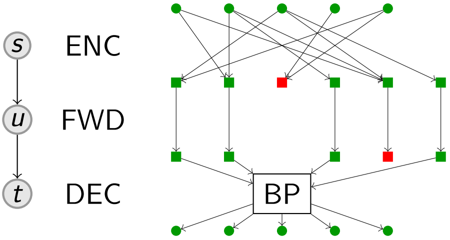
Forward error correction

- Example: fountain codes
- Scalable for multicast
- Neglectable feedback cost



Complexity of Fountain Codes with Routing

- K packets, T symbols in a packet.
- Encoding: $\mathcal{O}(T)$ per packet.
- Decoding: $\mathcal{O}(T)$ per packet.
- Routing: $\mathcal{O}(1)$ per packet and fixed buffer size.



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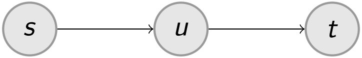
[Shokr06] A. Shokrollahi, "Raptor codes," IEEE Trans. Inform. Theory, vol. 52, no. 6, pp. 2551-2567, Jun 2006.

Achievable Rates

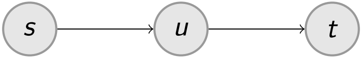


Both links have a packet loss rate 0.2.

The capacity of this network is 0.8.

Intermediate	End-to-End	Maximum Rate
forwarding	retransmission	0.64
forwarding	fountain codes	0.64

Achievable Rates



Both links have a packet loss rate 0.2.

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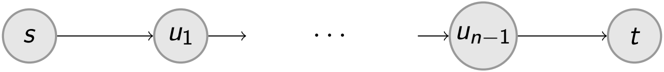
Intermediate	End-to-End	Maximum Rate
forwarding	retransmission	0.64
forwarding	fountain codes	0.64
network coding	random linear codes	0.8

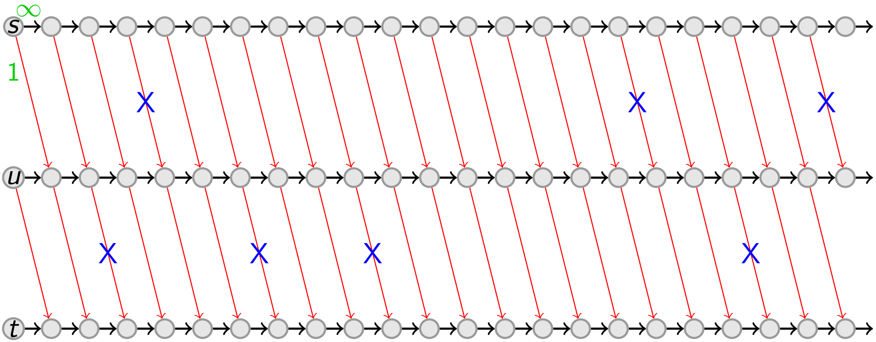
Achievable Rates: n hops



All links have a packet loss rate 0.2.

Intermediate Operation	Maximum Rate
forwarding	$0.8^n \to 0,\ n \to \infty$
network coding	0.8

An Explanation



Multicast capacity of erasure networks

Theorem

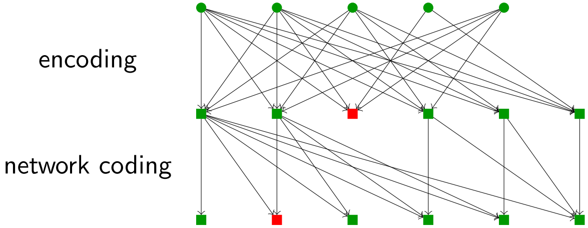
Random linear network codes achieve the capacity of a large range of multicast erasure networks.

[Wu06] Y. Wu, "A trellis connectivity analysis of random linear network coding with buffering," in Proc. IEEE ISIT 06, Seattle, USA, Jul. 2006.

LMKE08] D. S. Lun, M. Médard, R. Koetter, and M. Effros, "On coding for reliable communication over packet networks," Physical Communication, vol. 1, no. 1, pp. 320, 2008.

Complexity of Linear Network Coding

- Encoding: $\mathcal{O}(TK)$ per packet.
- Decoding: $\mathcal{O}(K^2 + TK)$ per packet.
- Network coding: $\mathcal{O}(TK)$ per packet. Buffer K packets.



Quick Summary

Routing + fountain

- ☺ low complexity
- ☹ low rate

Network coding

- ☹ high complexity
- ☺ high rate

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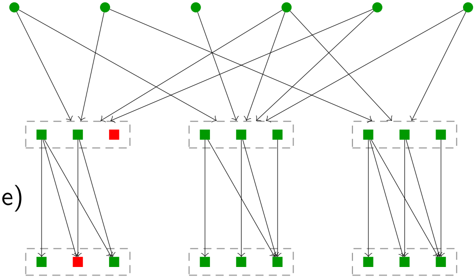
Batched Sparse (BATS) Codes

outer code

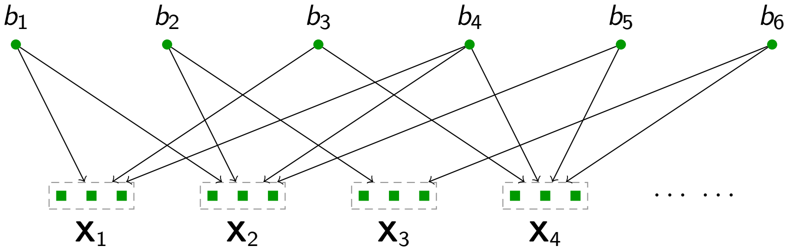
inner code (network code)



[YY11] S. Yang and R. W. Yeung. Coding for a network coded fountain. ISIT 2011, Saint Petersburg, Russia, 2011.

Encoding of BATS Code: Outer Code

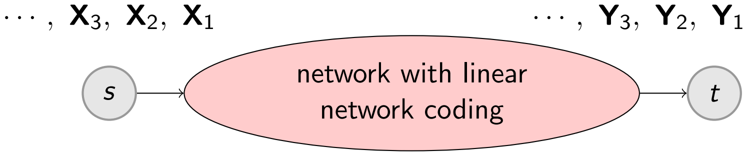
- Apply a "matrix fountain code" at the source node:
  1. Obtain a degree d by sampling a degree distribution Ψ.
  2. Pick d distinct input packets randomly.
  3. Generate a batch of M coded packets using the d packets.
- Transmit the batches sequentially.



$$\mathbf{X}_i = \begin{bmatrix} b_{i1} & b_{i2} & \cdots & b_{id_i} \end{bmatrix} \mathbf{G}_i = \mathbf{B}_i \mathbf{G}_i.$$

Encoding of BATS Code: Inner Code

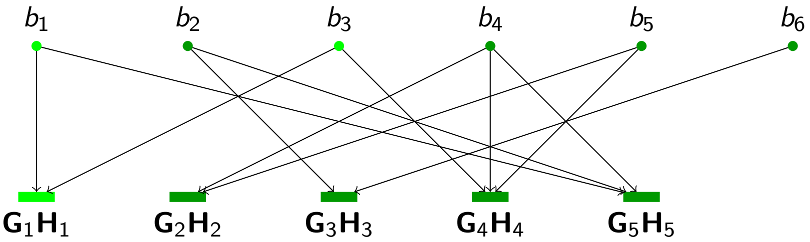
- The batches traverse the network.
- Encoding at the intermediate nodes forms the inner code.
- Linear network coding is applied in a causal manner within a batch.



$$\mathbf{Y}_i = \mathbf{X}_i \mathbf{H}_i, \ i = 1, 2, \ldots.$$

Belief Propagation Decoding

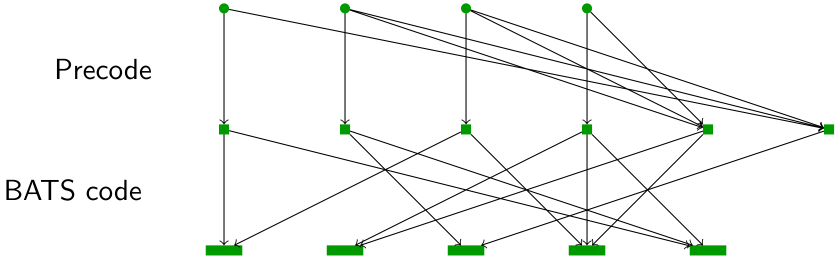
1. Find a check node i with $\text{degree}_i = \text{rank}(\mathbf{G}_i\mathbf{H}_i)$.
2. Decode the ith batch.
3. Update the decoding graph. Repeat 1).



The linear equation associated with a check node: $\mathbf{Y}_i = \mathbf{B}_i\mathbf{G}_i\mathbf{H}_i$.

Precoding

- Precoding by a fixed-rate erasure correction code.
- The BATS code recovers $(1-\eta)$ of its input packets.



[Shokr06] A. Shokrollahi, Raptor codes, IEEE Trans. Inform. Theory, vol. 52, no. 6, pp. 25512567, Jun. 2006.

Degree Distribution

We need a degree distribution Ψ such that

1. The BP decoding succeeds with high probability.
2. The encoding/decoding complexity is low.
3. The coding rate is high.

A Sufficient Condition

Define

$$\Omega(x) = \sum_{r=1}^{M} h^*_{r,r} \sum_{d=r+1}^{D} d\Psi_d I_{d-r,r}(x) + \sum_{r=1}^{M} h_{r,r} r \Psi_r,$$

where $h^*_{r,r}$ is related to the rank distribution of H and $\mathrm{I}_{a,b}(x)$ is the *regularized incomplete beta function*.

Theorem

Consider a sequence of decoding graph BATS$(K, n, \{\Psi_{d,r}\})$ *with constant* $\theta = K/n$. *The BP decoder is asymptotically error free if the degree distribution satisfies*

$$\Omega(x) + \theta \ln(1-x) > 0 \quad \text{for } x \in (0, 1-\eta),$$

An Optimization Problem

$$\begin{aligned} \max \quad & \theta \\ \text{s.t.} \quad & \Omega(x) + \theta \ln(1-x) \geq 0, \quad 0 < x < 1-\eta \\ & \Psi_d \geq 0, \quad d = 1, \cdots, D \\ & \sum_d \Psi_d = 1. \end{aligned}$$

- $D = \lceil M/\eta \rceil$
- Solver: Linear programming by sampling some x.

Complexity of Sequential Scheduling

<table>
<tr><td colspan="2">Source node encoding</td><td>$\mathcal{O}(TM)$ per packet</td></tr>
<tr><td colspan="2">Destination node decoding</td><td>$\mathcal{O}(M^2 + TM)$ per packet</td></tr>
<tr><td rowspan="2">Intermediate Node</td><td>buffer</td><td>$\mathcal{O}(TM)$</td></tr>
<tr><td>network coding</td><td>$\mathcal{O}(TM)$ per packet</td></tr>
</table>

T: length of a packet

K: number of packets

M: batch size

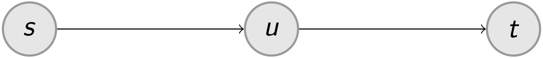
Achievable Rates

Optimization

$$
\begin{aligned}
\max \quad & \theta \\
\text{s.t.} \quad & \Omega(x_k) + \theta \ln(1 - x_k) \geq 0, \quad x_k \in (0, 1-\eta) \\
& \Psi_d \geq 0, \quad d = 1, \cdots, \lceil M/\eta \rceil \\
& \sum_d \Psi_d = 1.
\end{aligned}
$$

- The optimal values of θ is very close to $\mathrm{E}[\mathrm{rank}(H)]$.
- It can be proved when $\mathrm{E}[\mathrm{rank}(H)] = M \Pr\{\mathrm{rank}(H) = M\}$.

Simulation Result



- Packet loss rate 0.2.
- Node s encodes K packets using a BATS code.
- Node u caches only one batch.
- Node t sends one feedback after decoding.

Coding rates obtained by simulation for $M = 32$

K	$q = 2$	$q = 4$	$q = 8$	$q = 16$
16000	0.5826	0.6145	0.6203	0.6248
32000	0.6087	**0.6441**	**0.6524**	**0.6574**
64000	0.6259	**0.6655**	**0.6762**	**0.6818**

M: batch size

K: number of packets

q: field size

Tradeoff

- $M = 1$: BATS codes degenerate to Raptor codes.
  - Low complexity
  - No benefit of network coding
- $M = K$ and $\mathrm{degree} \equiv K$: BATS codes becomes RLNC.
  - High complexity
  - Full benefit of network coding.
- Exist parameters with moderate values that give very good performance

Outline

Recent Developments

- Degree distribution optimization
  - Degree distribution depends on the rank distribution.
  - Robust degree distribution for different rank distributions.
  - Inactivation decoding alleviates the degree distribution optimization problem.
- Finite length analysis [3]
- Testing systems
  - Multi-hop wireless transmission: 802.11
  - Peer-to-peer file transmission

Summary

- BATS codes provide a digital fountain solution with linear network coding:
  - Outer code at the source node is a matrix fountain code
  - Linear network coding at the intermediate nodes forms the inner code
  - Prevents BOTH packet loss and delay from accumulating along the way
- The more hops between the source node and the sink node, the larger the benefit.
- Future work:
  - Proof of (nearly) capacity achieving
  - Design of intermediate operations to maximize the throughput and minimize the buffer size

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