Communication Theory Workshop

Joint Space-Division and Multiplexing: How to Achieve Massive MIMO Gains in FDD Systems

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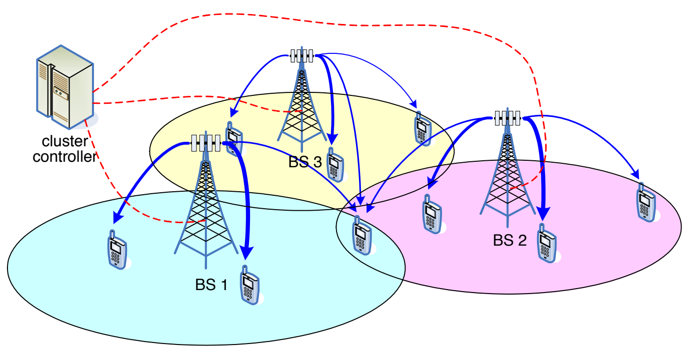
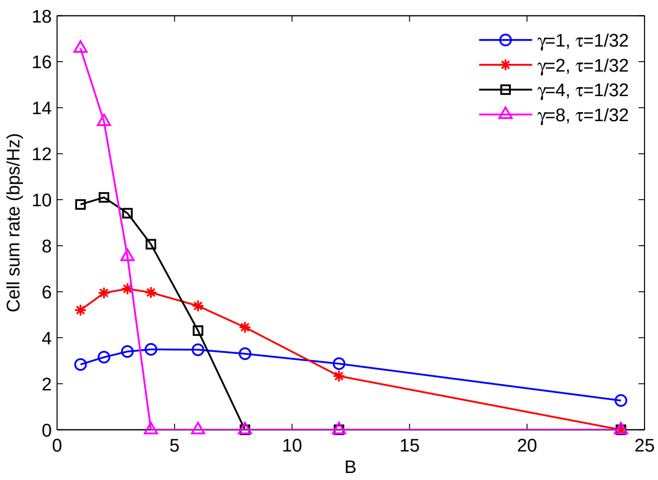
Channel estimation bottleneck on MU-MIMO

- High-SNR capacity of $N_t \times N_r$ single-user MIMO with coherence block-length T [Zheng-Tse, 2003]:

$$C(\mathsf{SNR}) = M^*(1 - M^*/T)\log \mathsf{SNR} + O(1), \qquad M^* = \min\{N_t, N_r, T/2\}$$

- Trivial cooperative bound: for large $M = N_t$ and $N = KN_r$, the coherence block T is the limiting factor.
- $\Rightarrow$ Disappointing theoretical performance of "CoMP" (base station cooperation), in FDD.





Channel model with antenna correlation

- In FDD, for large macro-cellular base stations, we have to exploit channel dimensionality reduction while still exploiting the large number of antennas at the BS.

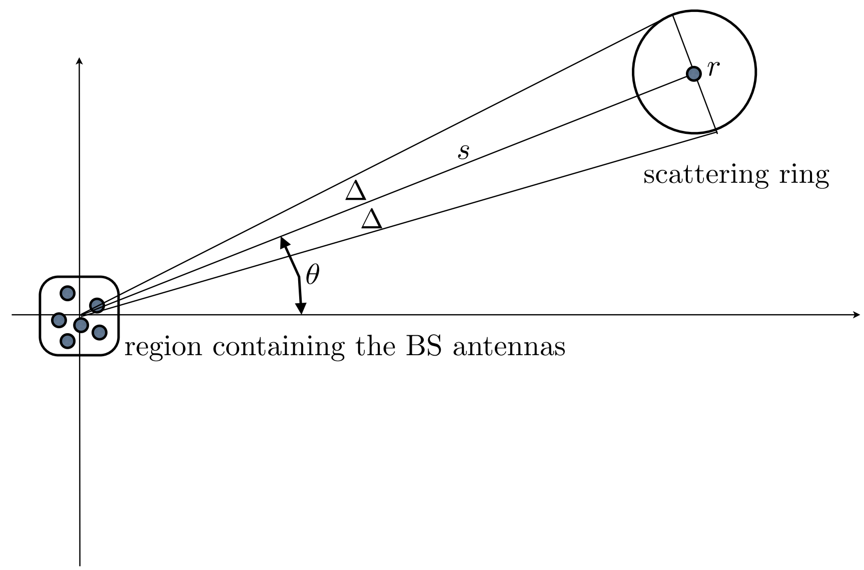
- Idea: exploit the asymmetric spatial channel correlation at the BS and at the UTs.

- Isotropic scattering, $|\mathbf{u} - \mathbf{u}'| = \lambda D$:

$$\mathbb{E}\left[h(\mathbf{u})h^*(\mathbf{u}')\right] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-j2\pi D\cos(\alpha)} d\alpha = J_0(2\pi D)$$

- Two users separated by a few meters (say 10 λ) are practically uncorrelated.

- In contrast, the base station sees user groups at different AoAs under narrow AS $\Delta \approx \arctan(\mathsf{r}/\mathsf{s})$.



- This leads to the Tx antenna correlation model

$$\mathbf{h} = \mathbf{U}\boldsymbol{\Lambda}^{1/2}\mathbf{w}, \quad \mathbf{R} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{U}^{\mathsf{H}}$$

with

$$[\mathbf{R}]_{m,p} = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{j\mathbf{k}^{\mathsf{T}}(\alpha+\theta)(\mathbf{u}_m - \mathbf{u}_p)} d\alpha.$$

Joint Space Division and Multiplexing (JSDM)

- K users selected to form G groups, with $\approx$ same channel correlation.

$$\underline{\mathbf{H}} = [\mathbf{H}_1, \ldots, \mathbf{H}_G], \text{ with } \mathbf{H}_g = \mathbf{U}_g \mathbf{\Lambda}_g^{1/2} \mathbf{W}_g.$$

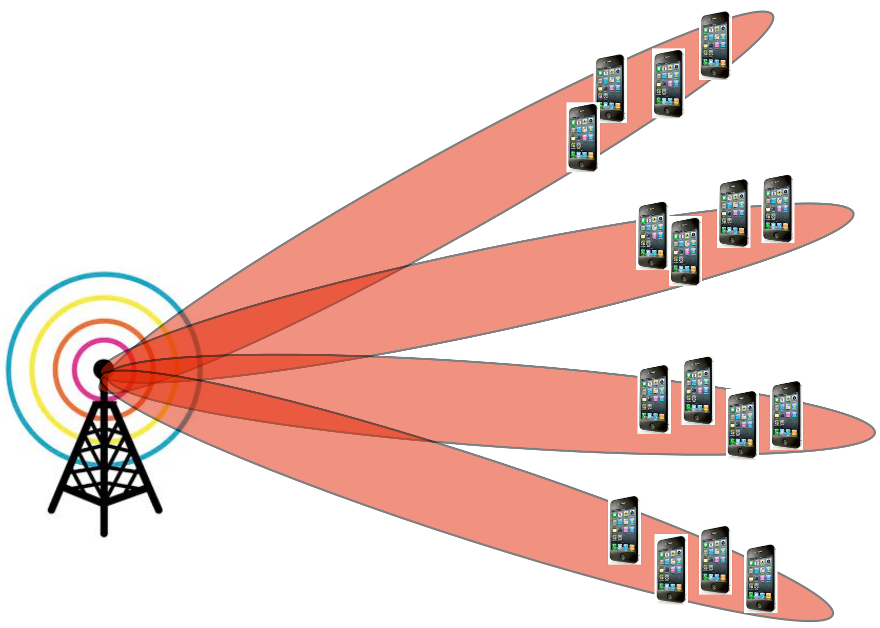
- Two-stage precoding: $\mathbf{V} = \mathbf{B}\mathbf{P}$.
- $\mathbf{B} \in \mathbb{C}^{M \times b_g}$ is a pre-beamforming matrix function of $\{\mathbf{U}_g, \mathbf{\Lambda}_g\}$ only.
- $\mathbf{P} \in \mathbb{C}^{b_g \times S_g}$ is a precoding matrix that depends on the effective channel.
- The effective channel matrix is given by

$$\underline{\mathbf{H}}^{\mathsf{H}} = \begin{bmatrix} \mathbf{H}_1^{\mathsf{H}}\mathbf{B}_1 & \mathbf{H}_1^{\mathsf{H}}\mathbf{B}_2 & \cdots & \mathbf{H}_1^{\mathsf{H}}\mathbf{B}_G \\ \mathbf{H}_2^{\mathsf{H}}\mathbf{B}_1 & \mathbf{H}_2^{\mathsf{H}}\mathbf{B}_2 & \cdots & \mathbf{H}_2^{\mathsf{H}}\mathbf{B}_G \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{H}_G^{\mathsf{H}}\mathbf{B}_1 & \mathbf{H}_G^{\mathsf{H}}\mathbf{B}_2 & \cdots & \mathbf{H}_G^{\mathsf{H}}\mathbf{B}_G \end{bmatrix}.$$

- Per-Group Processing: If estimation and feedback of the whole $\underline{\mathbf{H}}$ is still too costly, then each group estimates its own diagonal block $\mathbf{H}_g = \mathbf{B}_g^{\mathsf{H}}\mathbf{H}_g$, and $\mathbf{P} = \mathrm{diag}(\mathbf{P}_1, \cdots, \mathbf{P}_G)$.

- This results in

$$\mathbf{y}_g = \mathbf{H}_g^{\mathsf{H}}\mathbf{B}_g\mathbf{P}_g\mathbf{d}_g + \sum_{g' \neq g} \mathbf{H}_g^{\mathsf{H}}\mathbf{B}_{g'}\mathbf{P}_{g'}\mathbf{d}_{g'} + \mathbf{z}_g$$



Achieving capacity with reduced CSIT

- Let $r = \sum_{g=1}^{G} r_g$ and suppose that the channel covariances of the G groups are such that $\underline{\mathbf{U}} = [\mathbf{U}_1, \cdots, \mathbf{U}_G]$ is $M \times r$ *tall unitary* (i.e., $r \leq M$ and $\underline{\mathbf{U}}^{\mathsf{H}} \underline{\mathbf{U}} = \mathbf{I}_r$).

- Eigen-beamforming (let $b_g = r_g$ and $\mathbf{B}_g = \mathbf{U}_g$) achieves exact block diagonalization.

- The decoupled MU-MIMO channel takes on the form

$$\mathbf{y}_g = \mathbf{H}_g{}^{\mathsf{H}} \mathbf{P}_g \mathbf{d}_g + \mathbf{z}_g = \mathbf{W}_g^{\mathsf{H}} \Lambda_g^{1/2} \mathbf{P}_g \mathbf{d}_g + \mathbf{z}_g, \qquad \text{for } g = 1, \ldots, G,$$

where $\mathbf{W}_g$ is a $r_g \times K_g$ i.i.d. matrix with elements $\sim \mathcal{CN}(0, 1)$.

Theorem 1. *For $\underline{\mathbf{U}}$ tall unitary, JSDM with PGP achieves the same sum capacity of the corresponding MU-MIMO downlink channel with full CSIT.* ■

Block Diagonalization

- For given target numbers of streams per group $\{S_g\}$ and dimensions $\{b_g\}$ satisfying $S_g \leq b_g \leq r_g$, we can find the pre-beamforming matrices $\mathbf{B}_g$ such that:

$$\mathbf{U}_{g'}^{\mathsf{H}}\mathbf{B}_g = \mathbf{0} \quad \forall \;\; g' \neq g, \quad \text{and} \;\; \mathrm{rank}(\mathbf{U}_g^{\mathsf{H}}\mathbf{B}_g) \geq S_g$$

- Necessary condition for exact BD

$$\mathrm{Span}(\mathbf{B}_g) \subseteq \mathrm{Span}^{\perp}(\{\mathbf{U}_{g'} : g' \neq g\}).$$

- When $\mathrm{Span}^{\perp}(\{\mathbf{U}_{g'} : g' \neq g\})$ has dimension smaller than S_g, the rank condition on the diagonal blocks cannot be satisfied.

- In this case, S_g should be reduced (reduce the number of served users per group) or, as an alternative, approximated BD based on selecting $r_g^\star < r_g$ dominant eigenmodes for each group g can be implemented.

Performance analysis with regularized ZF

- The transformed channel matrix $\underline{\mathbf{H}}$ has dimension $b \times S$, with blocks $\mathbf{H}_g$ of dimension $b_g \times S_g$.

- For simplicity we allocate to all users the same fraction of the total transmit power, $p_{g_k} = \frac{P}{S}$.

- For PGP, the regularized zero forcing (RZF) precoding matrix for group g is given by

$$\mathbf{P}_{g,\mathrm{rzf}} = \bar{\zeta}_g \bar{\mathbf{K}}_g \mathbf{H}_g,$$

where

$$\bar{\mathbf{K}}_g = \left[\mathbf{H}_g \mathbf{H}_g^{\mathsf{H}} + b_g \alpha \mathbf{I}_{b_g}\right]^{-1}$$

and where

$$\bar{\zeta}_g^2 = \frac{S'}{\mathrm{tr}(\mathbf{H}_g^{\mathsf{H}} \mathbf{K}_g^{\mathsf{H}} \mathbf{B}_g^{\mathsf{H}} \mathbf{B}_g \mathbf{K}_g \mathbf{H}_g)}.$$

- The SINR of user g_k given by

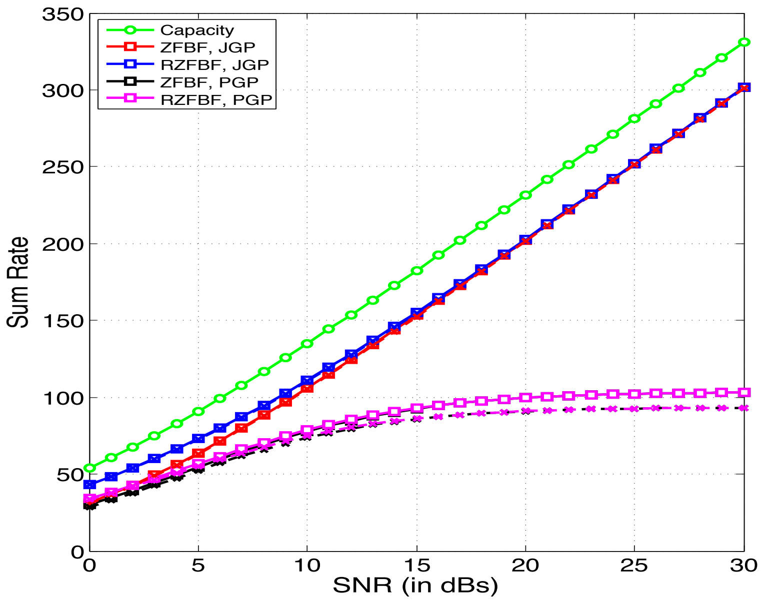
$$\gamma_{g_k,\mathrm{pgp}} = \frac{\frac{P}{S}\bar{\zeta}_g^2 |\mathbf{h}_{g_k}^{\mathsf{H}} \mathbf{B}_g \bar{\mathbf{K}}_g \mathbf{B}_g^{\mathsf{H}} \mathbf{h}_{g_k}|^2}{\frac{P}{S}\sum_{j\neq k} \bar{\zeta}_g^2 |\mathbf{h}_{g_k}^{\mathsf{H}} \mathbf{B}_g \bar{\mathbf{K}}_g \mathbf{B}_g^{\mathsf{H}} \mathbf{h}_{g_j}|^2 + \frac{P}{S}\sum_{g'\neq g}\sum_j \bar{\zeta}_{g'}^2 |\mathbf{h}_{g_k}^{\mathsf{H}} \mathbf{B}_{g'} \bar{\mathbf{K}}_{g'} \mathbf{B}_{g'}^{\mathsf{H}} \mathbf{h}_{g'_j}|^2 + 1}$$

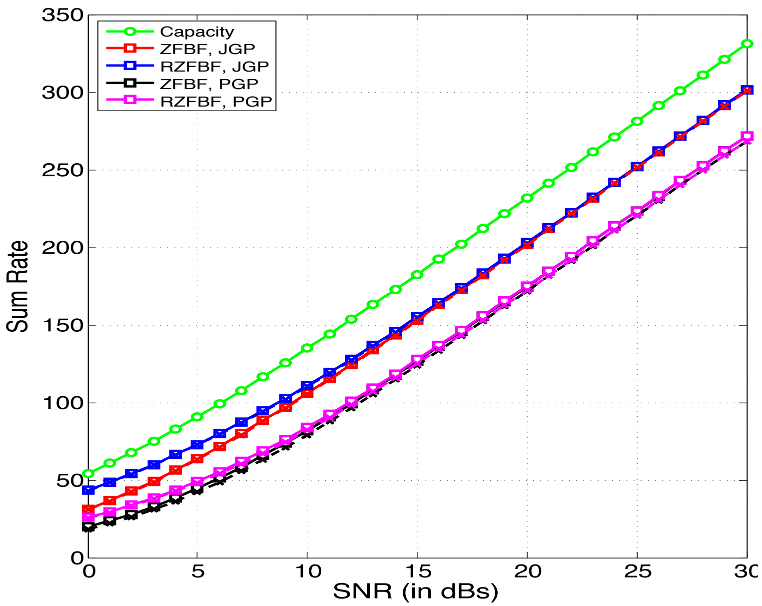
- Using the "deterministic equivalent" method of [Wagner, Couillet, Debbah, Slock, 2011], we can calculate $\gamma_{g_k,\mathrm{pgp}}^o$ such that

$$\gamma_{g_k,\mathrm{pgp}} - \gamma_{g_k,\mathrm{pgp}}^o \xrightarrow{M\to\infty} 0$$

Example

- $M = 100$, $G = 6$ user groups, Rank$(\mathbf{R}_g) = 21$, effective rank $r_g^* = 11$.
- We serve $S' = 5$ users per group with $b' = 10$, $r^\star = 6$ and $r^\star = 12$.
- For $r_g^* = 12$: 150 bit/s/Hz at SNR $= 18$ dB: 5 bit/s/Hz per user, for 30 users served simultaneously on the same time-frequency slot.





Training, Feedback and Computations Requirements

- Full CSI: 100×30 channel matrix $\Rightarrow$ 3000 complex channel coefficients per coherence block (CSI feedback), with 100×100 unitary "common" pilot matrix for downlink channel estimation.

- JSDM with PGP: $6 \times 10 \times 5$ diagonal blocks $\Rightarrow$ 300 complex channel coefficients per coherence block (CSI feedback), with 10×10 unitary "dedicated" pilot matrices for downlink channel estimation, sent in parallel to each group through the pre-beamforming matrix.

- One order of magnitude saving in both downlink training and CSI feedback.

- Computation: 6 matrix inversions of dimension 5×5, with respect to one matrix inversion of dimension 30×30.

Non-ideal CSIT

- Parallel downlink training in all groups: a scaled unitary training matrix $\mathbf{X}_{\rm tr}$ of dimension $b' \times b'$ is sent, simultaneously, to all groups in the common downlink training phase.

- Received signal at group g receivers is given by

$$\mathbf{Y}_g = \mathbf{H}_g^{\sf H} \mathbf{X}_{\rm tr} + \sum_{g' \neq g} \mathbf{H}_g{}^{\sf H} \mathbf{B}_{g'} \mathbf{X}_{\rm tr} + \mathbf{Z}_g.$$

- Multiplying from the right by $\mathbf{X}_{\rm tr}^{\sf H}$ and letting $\rho_{\rm tr}$ denote the power allocated to training, we obtain

$$\mathbf{Y}_g \mathbf{X}_{\rm tr}^{\sf H} = \rho_{\rm tr} \mathbf{H}_g^{\sf H} + \rho_{\rm tr} \sum_{g' \neq g} \mathbf{H}_g{}^{\sf H} \mathbf{B}_{g'} + \mathbf{Z}_g \mathbf{X}_{\rm tr}^{\sf H}.$$

- The relevant observation for the g_k-th user effective channel is:

$$\widetilde{\mathbf{h}}_{g_k} = \sqrt{\rho_{\text{tr}}}\mathbf{h}_{g_k} + \sqrt{\rho_{\text{tr}}}\left(\sum_{g' \neq g} \mathbf{B}_{g'}^{\mathsf{H}}\right)\mathbf{h}_{g_k} + \widetilde{\mathbf{z}}_{g_k}.$$

- The corresponding MMSE estimator is given by

$$\begin{aligned}
\widehat{\mathbf{h}}_{g_k} &= \mathbb{E}\left[\mathbf{h}_{g_k}\widetilde{\mathbf{h}}_{g_k}^{\mathsf{H}}\right]\mathbb{E}\left[\widetilde{\mathbf{h}}_{g_k}\widetilde{\mathbf{h}}_{g_k}^{\mathsf{H}}\right]^{-1}\widetilde{\mathbf{h}}_{g_k} \\
&= \sqrt{\rho_{\text{tr}}}\left[\mathbf{B}_g^{\mathsf{H}}\mathbf{R}_g\sum_{g'=1}^{G}\mathbf{B}_{g'}\right]\left[\rho_{\text{tr}}\sum_{g',g''=1}^{G}\mathbf{B}_{g'}^{\mathsf{H}}\mathbf{R}_g\mathbf{B}_{g''} + \mathbf{I}_{b'}\right]^{-1}\widetilde{\mathbf{h}}_{g_k} \\
&= \frac{1}{\sqrt{\rho_{\text{tr}}}}\left(\mathbf{M}_g\tilde{\mathbf{R}}_g\mathbf{O}^{\mathsf{T}}\right)\left[\mathbf{O}\tilde{\mathbf{R}}_g\mathbf{O}^{\mathsf{T}} + \frac{1}{\rho_{\text{tr}}}\mathbf{I}_{b'}\right]^{-1}\widetilde{\mathbf{h}}_{g_k}
\end{aligned}$$

where we used the fact that $\mathbf{h}_{g_k} = \mathbf{B}_g^{\mathsf{H}} \mathbf{h}_{g_k}$, and we introduced the $b' \times b$ block matrices

$$\begin{aligned} \mathbf{M}_g &= [\mathbf{0}, \ldots, \mathbf{0}, \underbrace{\mathbf{I}_{b'}}_{\text{block } g}, \mathbf{0}, \ldots, \mathbf{0}] \\ \mathbf{O} &= [\mathbf{I}_{b'}, \mathbf{I}_{b'}, \ldots, \mathbf{I}_{b'}]. \end{aligned}$$

- Notice that in the case of perfect BD we have that $\mathbf{R}_g \mathbf{B}_{g'} = \mathbf{0}$ for $g' \neq g$. Therefore, the MMSE estimator reduces to

$$\widehat{\mathbf{h}}_{g_k} = \frac{1}{\sqrt{\rho_{\rm tr}}} \bar{\mathbf{R}}_g \left[\bar{\mathbf{R}}_g + \frac{1}{\rho_{\rm tr}} \mathbf{I}_{b'} \right]^{-1} \widetilde{\mathbf{h}}_{g_k}$$

where $\bar{\mathbf{R}}_g = \mathbf{B}_g^{\mathsf{H}} \mathbf{R}_g \mathbf{B}_g$.

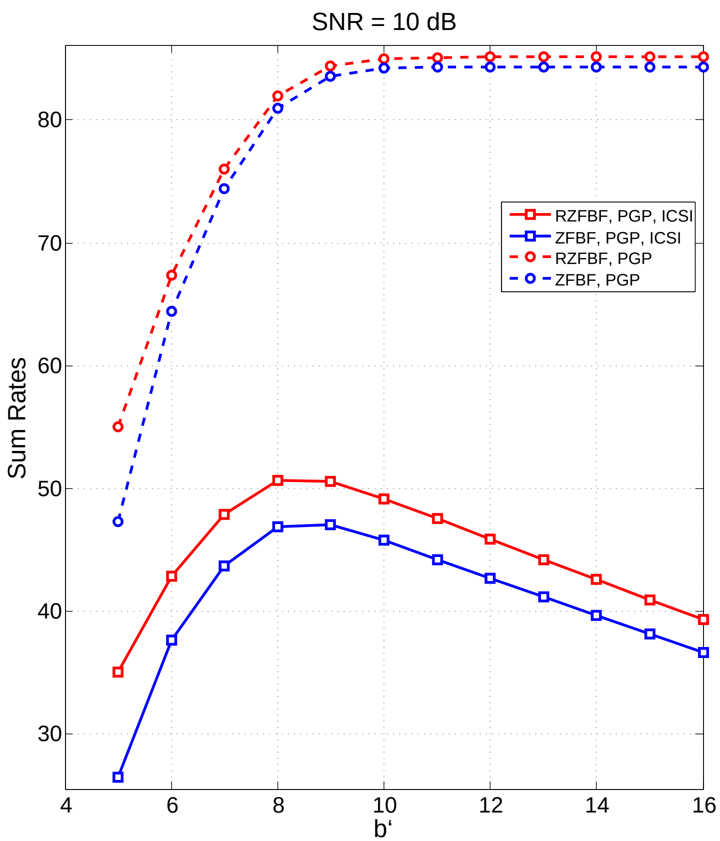
- Also in this case, the deterministic equivalent approximations of the SINR terms for RZFBF and ZFBF precoding can be be computed.

- Eventually, the achievable rate of user g_k is given by

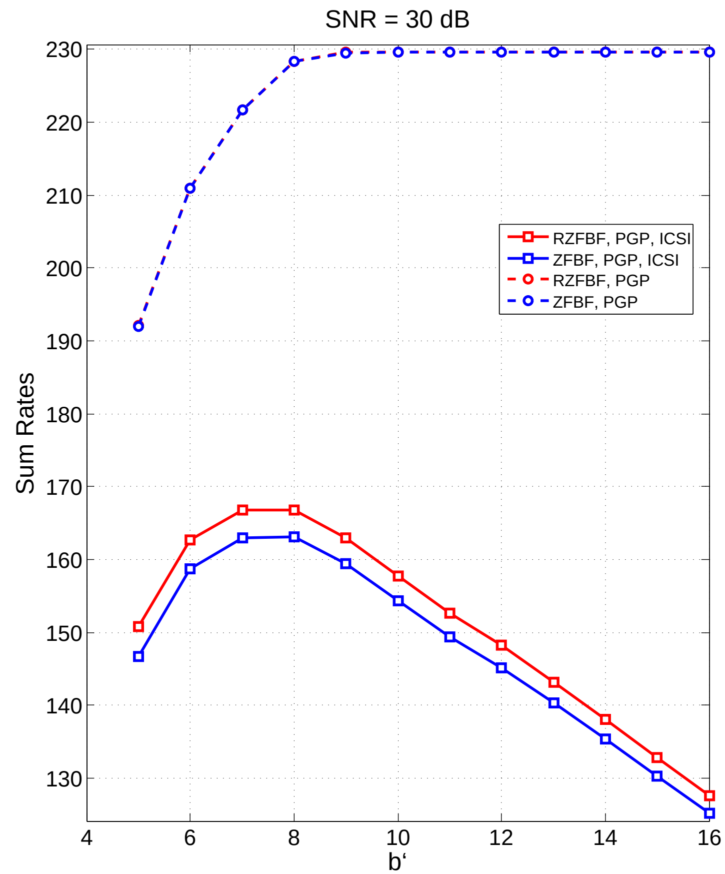
$$R_{g_k,\mathrm{pgp},\mathrm{csit}} = \max\left\{1 - \frac{b'}{T}, 0\right\} \times \log\left(1 + \widehat{\gamma}^o_{g_k,\mathrm{pgp},\mathrm{csit}}\right).$$

Tradeoff parameter b'

- b' large yields better conditioned matrices, but it “costs” more in terms of training phase dimension.

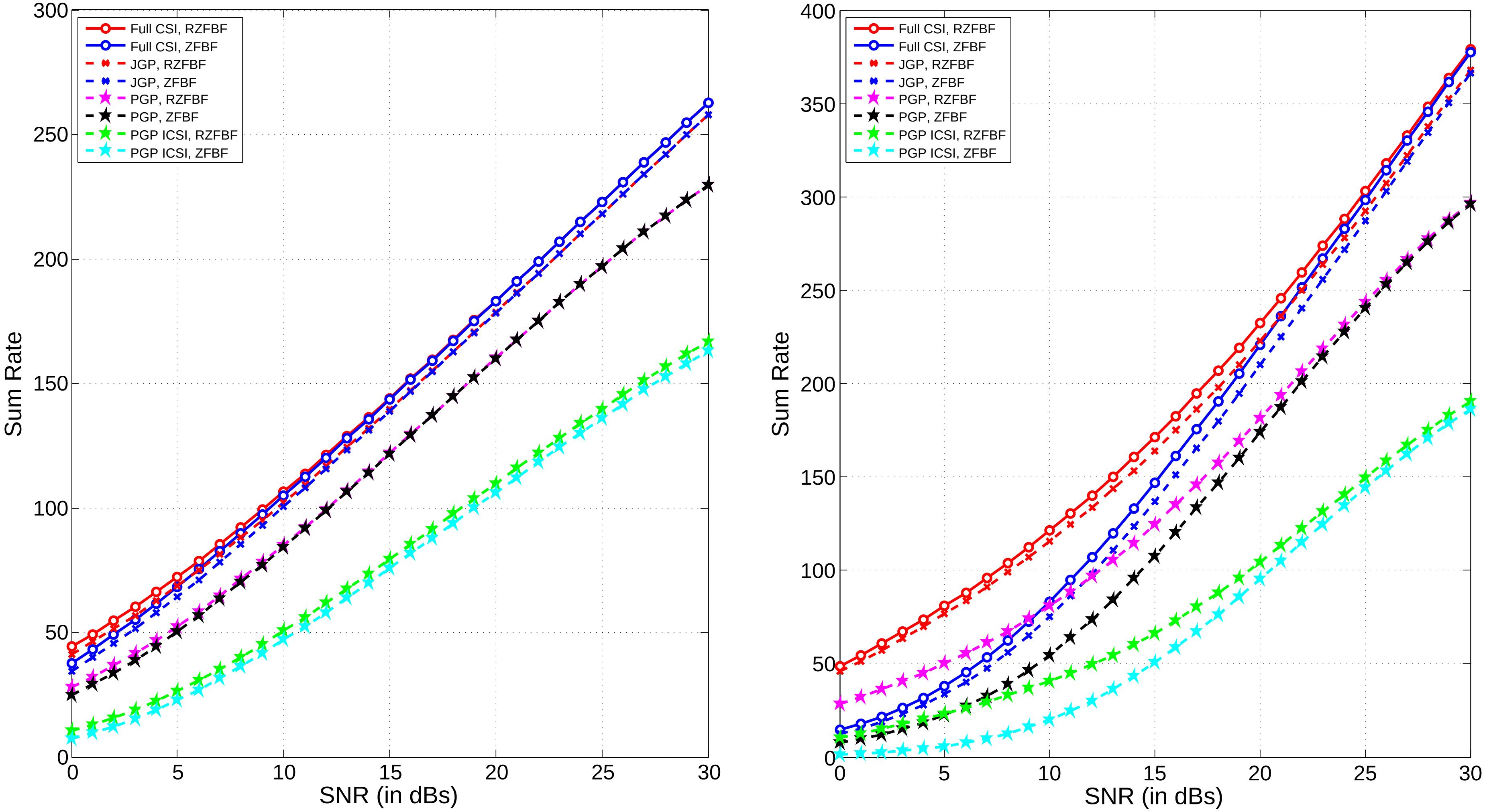


(a) S’ = 4, SNR = 10dB



(b) S’ = 8, SNR = 30dB

Impact of non-ideal CSIT



(c) S’ = 4

(d) S’ = 8

Discussion: is the tall unitary realistic?

- For a Uniform Linear Array (ULA), $\mathbf{R}$ is Toeplitz, with elements

$$[\mathbf{R}]_{m,p} = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{-j2\pi D(m-p)\sin(\alpha+\theta)} d\alpha, \quad m, p \in \{0, 1, \ldots, M-1\}$$

- We are interested in calculating the asymptotic rank, eigenvalue CDF and structure of the eigenvectors, for M large, for given geometry parameters D, θ, Δ.

- Correlation function

$$r_m = \frac{1}{2\Delta}\int_{-\Delta}^{\Delta} e^{-j2\pi Dm\sin(\alpha+\theta)} d\alpha.$$

- As $M \to \infty$, the eigenvalues of $\mathbf{R}$ tend to the "power spectral density" (i.e., the DT Fourier transform of r_m),

$$S(\xi) = \sum_{m=-\infty}^{\infty} r_m e^{-j2\pi\xi m}$$

sampled at $\xi = k/M$, for $k = 0, \ldots, M-1$.

- After some algebra, we arrive at

$$S(\xi) = \frac{1}{2\Delta} \sum_{m \in [D\sin(-\Delta+\theta)+\xi, D\sin(\Delta+\theta)+\xi]} \frac{1}{\sqrt{D^2 - (m-\xi)^2}}.$$

Szego's Theorem: eigenvalues

Theorem 2. *The empirical spectral distribution of the eigenvalues of* $\mathbf{R}$*,*

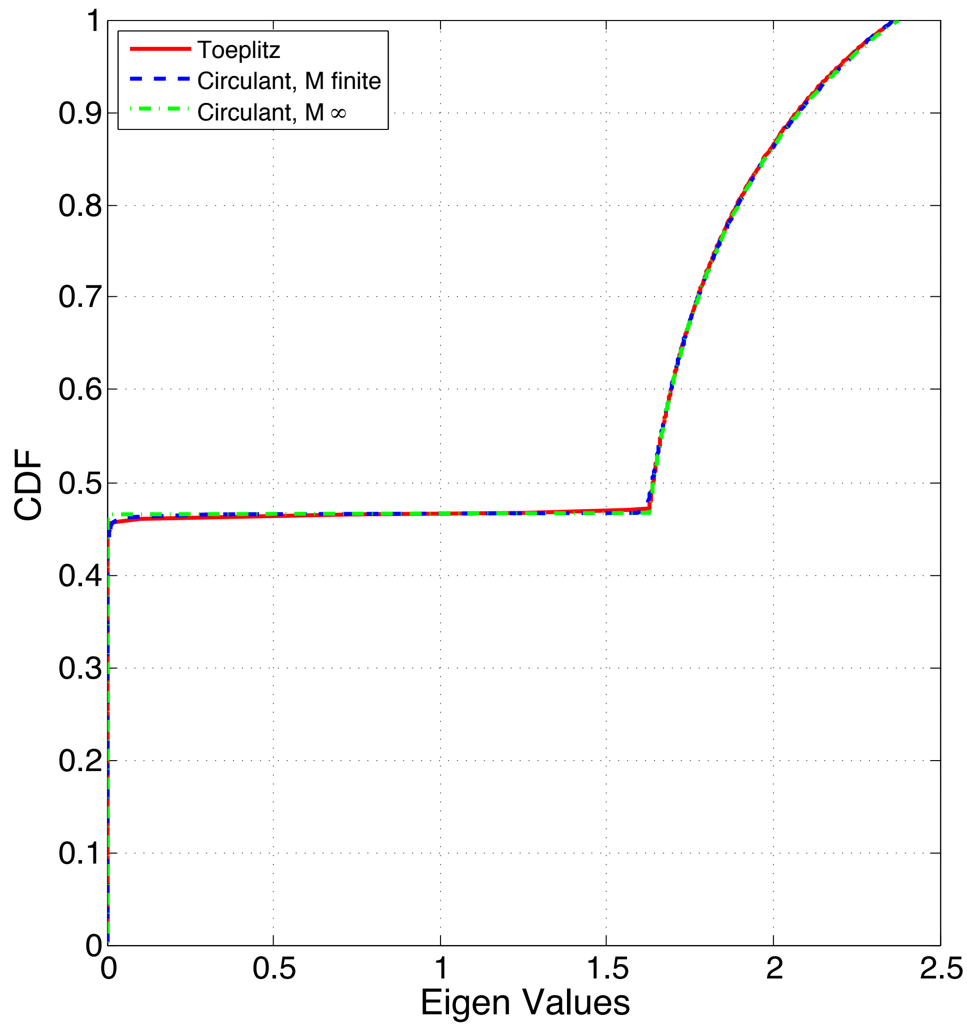
$$F_{\mathbf{R}}^{(M)}(\lambda) = \frac{1}{M}\sum_{m=1}^{M} 1\{\lambda_m(\mathbf{R}) \le \lambda\},$$

converges weakly to the limiting spectral distribution

$$\lim_{M\to\infty} F_{\mathbf{R}}^{(M)}(\lambda) = F(\lambda) = \int_{S(\xi)\le\lambda} d\xi.$$

■

Example: $M = 400, \theta = \pi/6, D = 1, \Delta = \pi/10$. Exact empirical eigenvalue cdf of $\mathbf{R}$ (red), its approximation the circulant matrix $\mathbf{C}$ (dashed blue) and its approximation from the samples of $S(\xi)$ (dashed green).



A less well-known Szego's Theorem: eigenvectors

Theorem 3. *Let* $\lambda_0(\mathbf{R}) \leq \ldots, \leq \lambda_{M-1}(\mathbf{R})$ *and* $\lambda_0(\mathbf{C}) \leq \ldots, \leq \lambda_{M-1}(\mathbf{C})$ *denote the set of ordered eigenvalues of* $\mathbf{R}$ *and* $\mathbf{C}$*, and let* $\mathbf{U} = [\mathbf{u}_0, \ldots, \mathbf{u}_{M-1}]$ *and* $\mathbf{F} = [\mathbf{f}_0, \ldots, \mathbf{f}_{M-1}]$ *denote the corresponding eigenvectors. For any interval* $[a, b] \subseteq [\kappa_1, \kappa_2]$ *such that* $F(\lambda)$ *is continuous on* $[a, b]$*, consider the eigenvalues index sets* $\mathcal{I}_{[a,b]} = \{m : \lambda_m(\mathbf{R}) \in [a, b]\}$ *and* $\mathcal{J}_{[a,b]} = \{m : \lambda_m(\mathbf{C}) \in [a, b]\}$*, and define* $\mathbf{U}_{[a,b]} = (\mathbf{u}_m : m \in \mathcal{I}_{[a,b]})$ *and* $\mathbf{F}_{[a,b]} = (\mathbf{f}_m : m \in \mathcal{J}_{[a,b]})$ *be the submatrices of* $\mathbf{U}$ *and* $\mathbf{F}$ *formed by the columns whose indices belong to the sets* $\mathcal{I}_{[a,b]}$ *and* $\mathcal{J}_{[a,b]}$*, respectively. Then, the eigenvectors of* $\mathbf{C}$ *approximate the eigenvectors of* $\mathbf{R}$ *in the sense that*

$$\lim_{M \to \infty} \frac{1}{M} \left\| \mathbf{U}_{[a,b]} \mathbf{U}_{[a,b]}^{\mathsf{H}} - \mathbf{F}_{[a,b]} \mathbf{F}_{[a,b]}^{\mathsf{H}} \right\|_F^2 = 0.$$

■

Consequence 1: $\mathbf{U}_g$ is well approximated by a "slice" of the DFT matrix.
Consequence 2: DFT pre-beamforming is near optimal for large M.

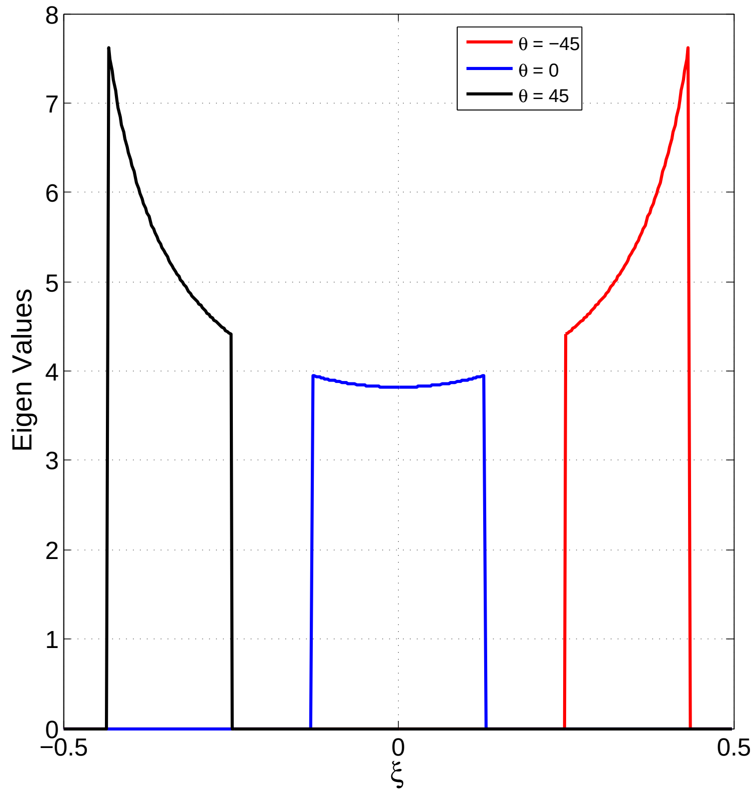
Theorem 4. *The asymptotic normalized rank of the channel covariance matrix* $\mathbf{R}$*, with antenna separation* λD*, AoA* θ *and AS* Δ*, is given by*

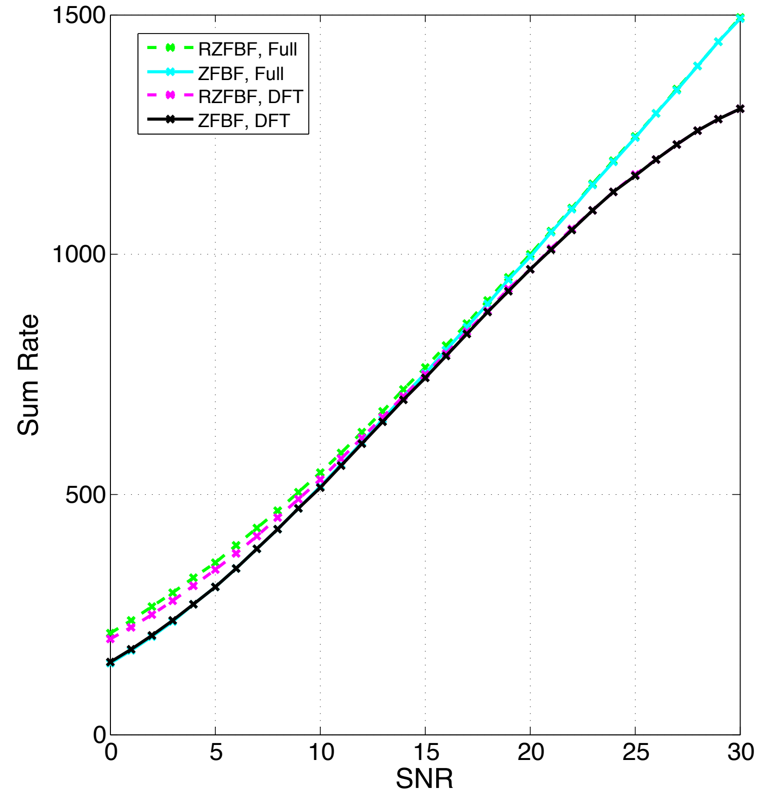
$$\rho = \min\{1, B(D, \theta, \Delta)\},$$

with $B(D, \theta, \Delta) = |D \sin(-\Delta + \theta) - D \sin(\Delta + \theta)|$*.* ■

Theorem 5. *Groups* g *and* g' *with angle of arrival* θ_g *and* $\theta_{g'}$ *and common angular spread* Δ *have spectra with disjoint support if their AoA intervals* $[\theta_g - \Delta, \theta_g + \Delta]$ *and* $[\theta_{g'} - \Delta, \theta_{g'} + \Delta]$ *are disjoint.* ■

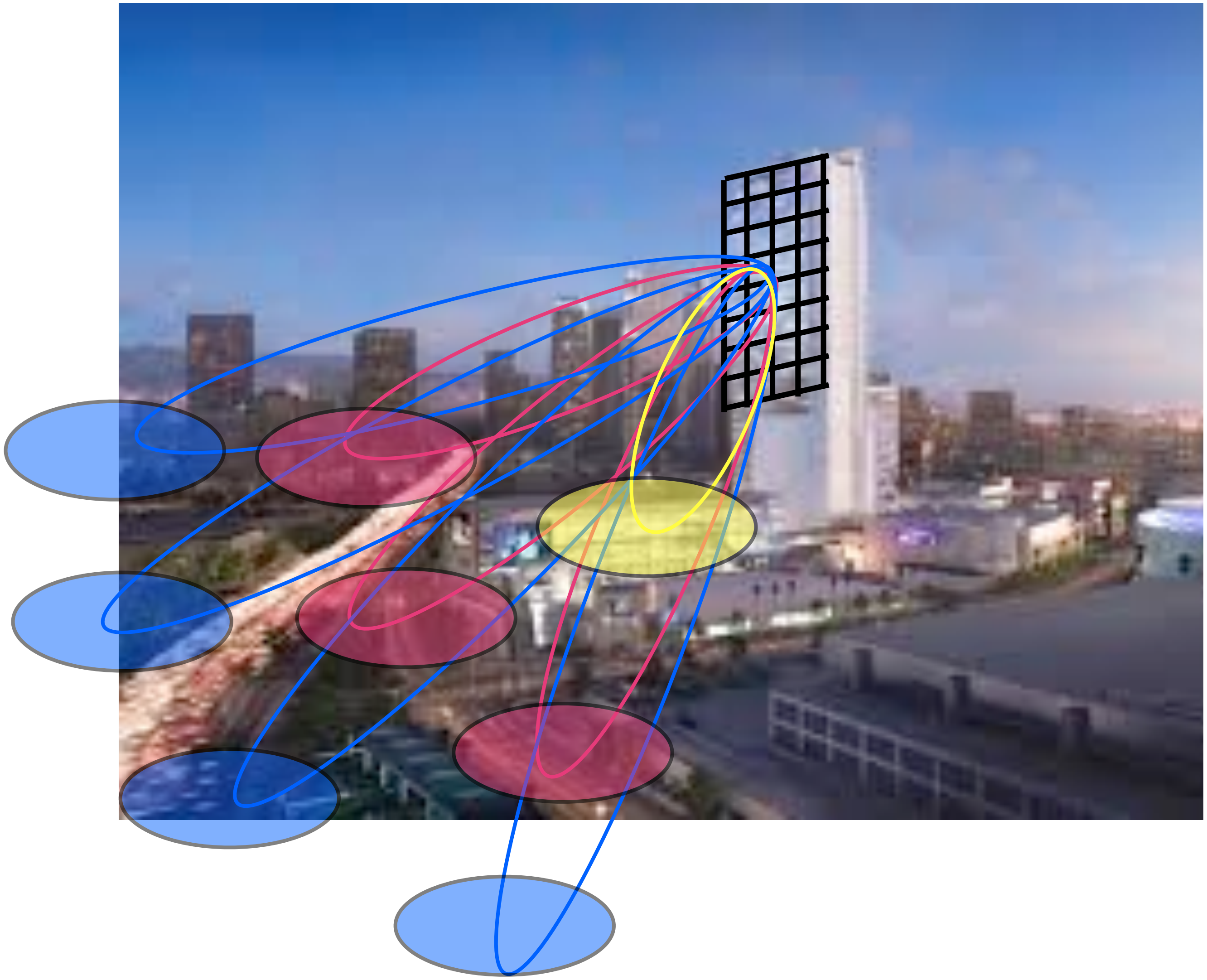
DFT Pre-Beamforming





- ULA with $M = 400$, $G = 3$, $\theta_1 = \frac{-\pi}{4}, \theta_2 = 0, \theta_3 = \frac{\pi}{4}$, $D = 1/2$ and $\Delta = 15$ deg.

Super-Massive MIMO



- Idea: produce a 3D pre-beamforming by Kronecker product of a “vertical” beamforming, separating the sector into L concentric regions, and a “horizontal” beamforming, separating each ℓ-th region into G_ℓ groups.

- Horizontal beam forming is as before.

- For vertical beam forming we just need to find one dominating eigenmode per region, and use the BD approach.

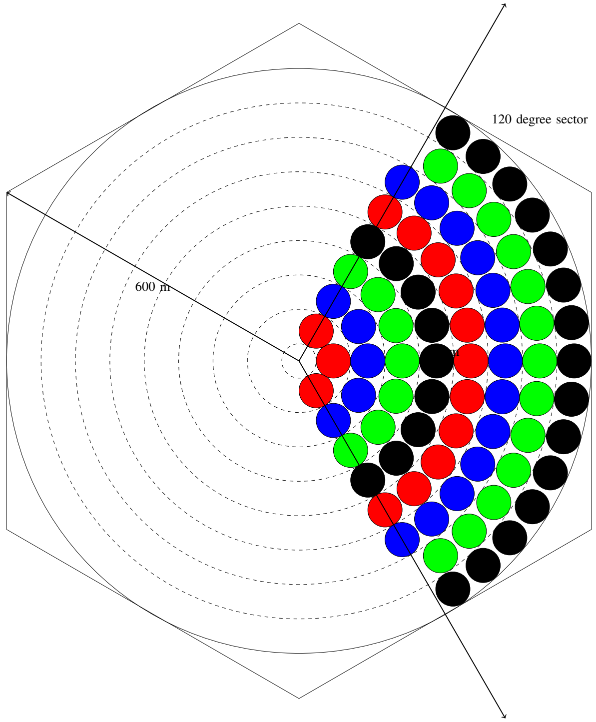
- A set of simultaneously served groups forms a “pattern”.

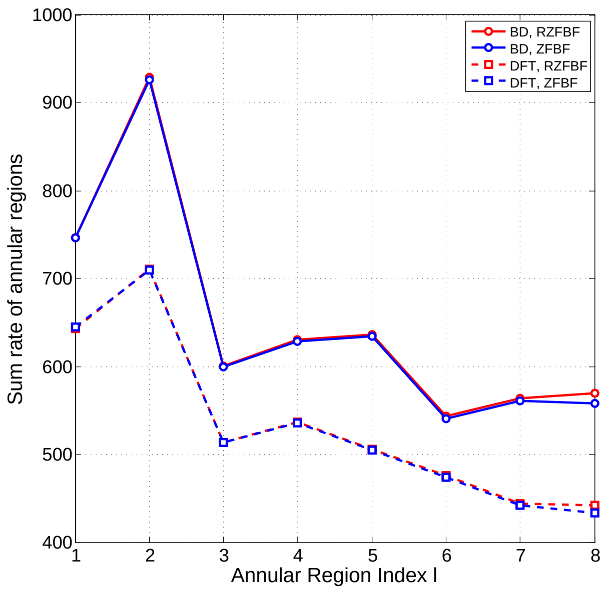
- Patterns need not cover the whole sector.

- Different intertwined patterns can be multiplexed in the time-frequency domain in order to guarantee a fair coverage.

An example

- Cell radius 600m, group ring radius 30m, array height 50m, $M = 200$ columns, $N = 300$ rows.
- Pathloss $g(x) = \frac{1}{1+(\frac{x}{d_0})^\delta}$ with $\delta = 3.8$ and $d_0 = 30$m.
- Same color regions are served simultaneously. Each ring is given equal power.





Sum throughput (bit/s/Hz) under PFS and Max-min Fairness

Scheme	Approximate BD	DFT based
PFS, RZFBF	1304.4611	1067.9604
PFS, ZFBF	1298.7944	1064.2678
MAXMIN, RZFBF	1273.7203	1042.1833
MAXMIN, ZFBF	1267.2368	1037.2915

1000 bit/s/Hz $\times$ 40 MHz of bandwidth $= 40$ Gb/s per sector.

Our on-going work

- Compatibility with an in-band Small-Cell tier: eICIC in the spatial domain: turn on and off the "spotbeams".
- Multi-cell strategies: activate mutually compatible patterns of groups in adjacent sectors.
- User grouping: we developed a very efficient way to cluster users according to their dominant subspaces (quantization according to chordal distance). See [Adhikary, Caire, arXiv:1305.7252].
- Hybrid Beamforming and mm-wave application: the DFT pre-beamforming can be implemented by phase shifters in analog domain.
- Estimation of the long-term channel statistics: revamped interest in super-resolution methods (MUSIC, ESPRIT) especially for the mm-wave case.

Conclusions

- Exploiting transmit antenna correlation reduces the channel to a simpler $\approx$ block diagonal structure.
- This is generalized sectorization! with MU-MIMO independently in each “sector” (group).
- We need only very coarse information on AoA and AS for the users .... DFT pre-beamforming.
- The idea can be easily extended to 3D beamforming (introducing elevation direction, Kronecker product structure).
- Downlink training, CSIT feedback and computation are greatly reduced (suitable for FDD).
- JSDM lends itself naturally to spatial-domain eICIC, simple inter-cell coordination, hybrid beamforming for mm-wave applications.

Thank You