On the Cost of CSI Acquisition in Large MIMO Systems

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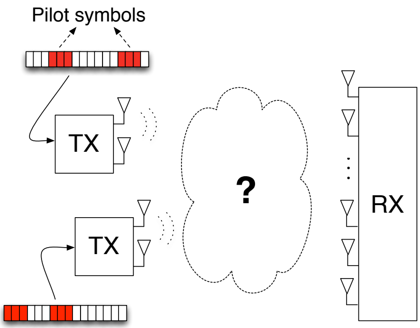
Joint work with **Wei Yang**, **Günther Koliander**, **Erwin Riegler**, **Franz Hlawatsch**, **Tobias Koch**, **Yury Polyanskiy**

Many thanks to **Ericsson Research Foundation**!

CSI acquisition limits large-MIMO gains



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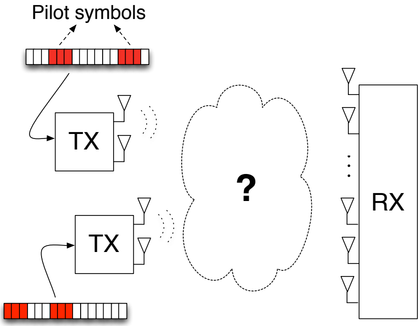
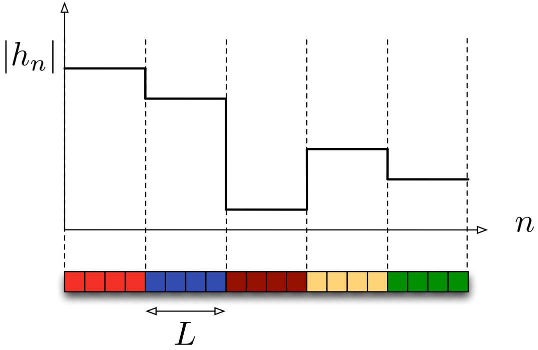


Capacity **in the absence of a priori** channel knowledge is the **ultimate limit** on the rate of reliable communication

Outline

1. Beyond the pre-log
2. Generic block-fading models
3. From asymptotics to finite-blocklength bounds

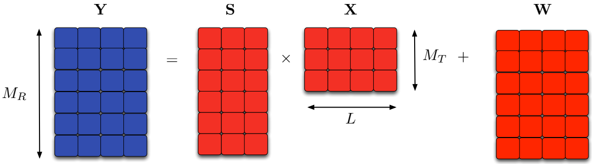
A simple channel model



Constant block-memoryless Rayleigh-fading channel

Coherence time is the bottleneck

- MIMO input-output relation



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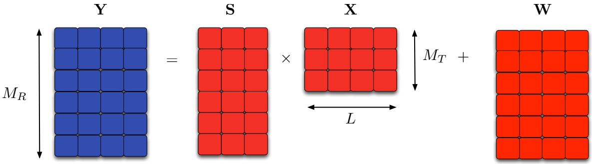

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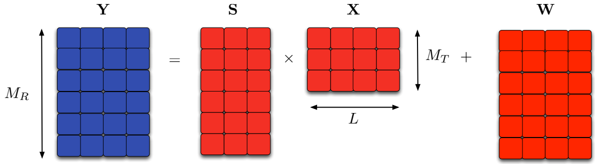
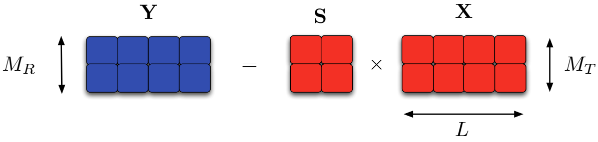


- No closed-form expression available for $C(\rho)$
- Pre-log [*Zheng & Tse, 2002*]

$$\chi = \lim_{\rho \to \infty} \frac{C(\rho)}{\log \rho} = M^* \left(1 - \frac{M^*}{L}\right)$$

where $M^* = \min\{M_T, M_R, L/2\}$

The underlying geometry: $M_T = M_R = M$



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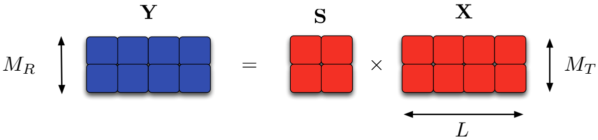

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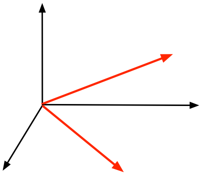

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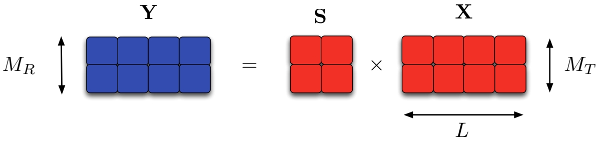

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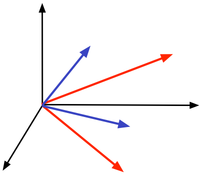

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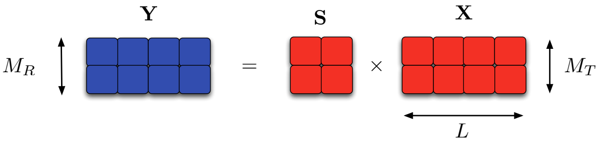

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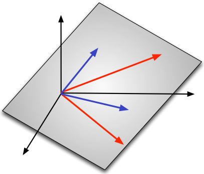
Communications on the Grassmannian manifold



Geometry suggests a signaling scheme

- **Uniform distribution** on the Grassmannian

$$\mathbf{X} = \sqrt{L\rho}\,\mathbf{U}$$

- $\mathbf{U}$: (truncated) **unitary** and **isotropically distributed**
- Unitary space-time modulation (**USTM**)

A conjecture

- Case $L \geq M_T + M_R$ ("small MIMO") [*Zheng & Tse (IT 2002)*]:

$$C(\rho) = R_{\mathrm{USTM}}(\rho) + o(1)$$

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- **Conjecture** for $L < M_T + M_R$ ("large MIMO") [*Zheng & Tse (IT 2002)*]:

  - USTM **not** $o(1)$**-optimal**

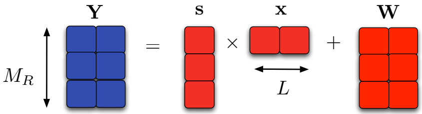
BSTM is the optimal distribution

[*Yang, Durisi, Riegler (JSAC 2013)*]

BSTM is $o(1)$-optimal when $L < M_T + M_R$ (large-MIMO)

- $\mathbf{X} = \mathbf{D}\mathbf{U}$ with
- $\mathbf{U}$ i.d. and unitary
- $\mathbf{D}^2$ diagonal; contains the eigenvalues of a **complex matrix-variate beta distributed** matrix

Why is BSTM optimal? The SIMO case



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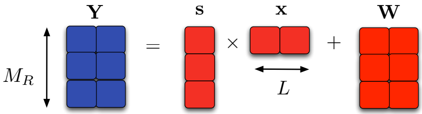

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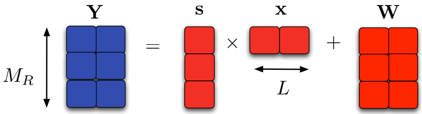

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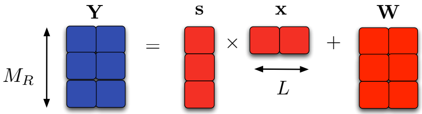

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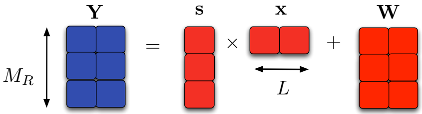

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$$I(\mathbf{x};\mathbf{Y}) = h(\mathbf{Y}) - h(\mathbf{Y}\,|\,\mathbf{x})$$

$$\approx h(\mathbf{s}\|\mathbf{x}\|) + 2(L - 1 - M_R)\,\mathbb{E}[\log\|\mathbf{x}\|] + \text{const}$$

Outline

The "generic" block-fading model

- Constant block-fading model for subchannel (r, t)

$$\mathbf{h}_{r,t} = \mathbf{1}_L \cdot s_{r,t}, \quad s_{r,t} \sim \mathcal{CN}(0, 1)$$

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- We assume that $\{\mathbf{z}_{r,t}\}$ are generic

Generic $\{\mathbf{z}_{r,t}\}$ yield larger pre-log

[*Riegler, Koliander, Durisi, Hlawatsch (ISIT 2013)*]

- $\{\mathbf{z}_{r,t}\}$ generic and $M_R > \frac{M_T(L-1)}{L-T}$ with $M_T < L/2$

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- Then

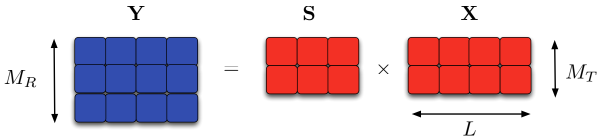
$$\chi_{\text{gen}} = M_T\left(1 - \frac{1}{L}\right)$$

- Compare with constant block-fading model

$$\chi_{\text{const}} = M_T\left(1 - \frac{M_T}{L}\right)$$

Intuition behind pre-log increase: $M_R = 3, M_T = 2, L = 4$

- Constant block-fading: $\chi_{\text{const}} = M_T \left(1 - \frac{M_T}{L}\right) = 1$



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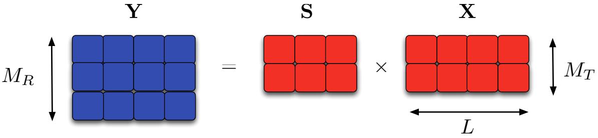
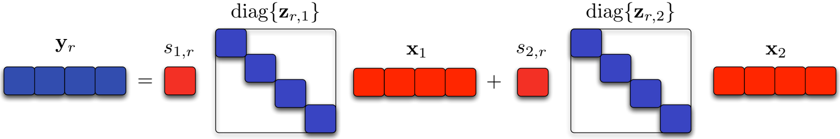


- Generic block-fading: $\chi_{\text{gen}} = M_T\left(1 - \frac{1}{L}\right) = \frac{3}{2}$



Outline

Lost in "asymptotia"?

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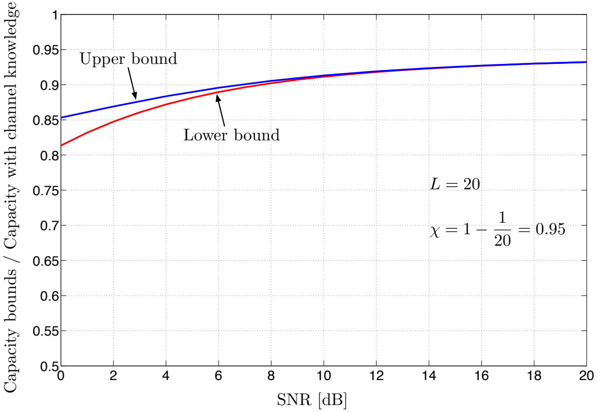
- capacity characterizations up to $o(1)$ yield tight bounds ☺
- pre-log sensitive to small changes in the channel model ☹

From asymptotia to tight bounds

[*Yang, Durisi, Koch, Polyanskiy (ITW 2012)*]



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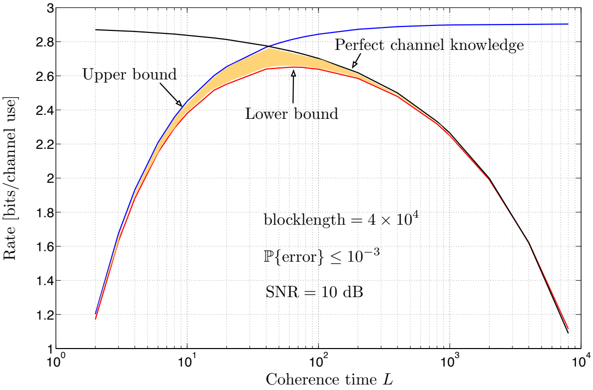

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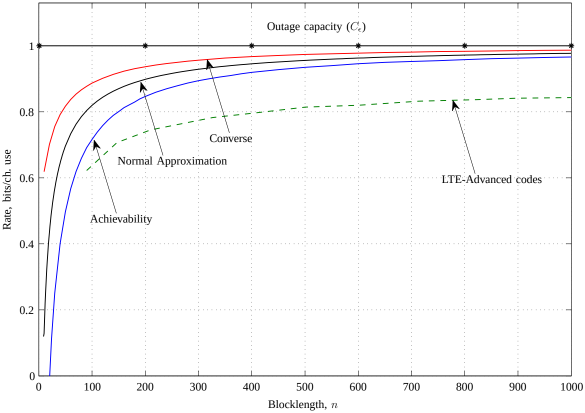

Zero dispersion

- AWGN channel [*Polyanskiy, Poor, Verdú (IT 2010)*]

$$R^*_{\mathrm{awgn}}(n,\epsilon) = C_{\mathrm{awgn}} - \sqrt{\frac{V}{n}}Q^{-1}(\epsilon) - \mathcal{O}\left(\frac{\log n}{n}\right)$$

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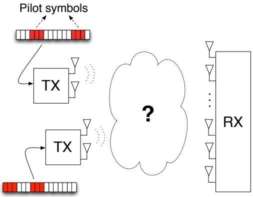
- SISO quasi static [*Yang, Durisi, Koch, Polyanskiy (ISIT 2013)*]

$$\{R^*_{\mathsf{csirt}}(n,\epsilon), R^*_{\mathsf{no}}(n,\epsilon)\} = C_\epsilon - 0\cancel{\sqrt{\frac{1}{n}}} - \mathcal{O}\left(\frac{\log n}{n}\right)$$

Summary

Capacity without a-priori CSI



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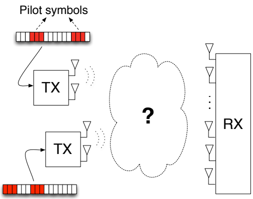

Too conservative estimates?

- USTM $\Rightarrow$ BSTM
- $M\left(1 - M/L\right) \Rightarrow M\left(1 - 1/L\right)$

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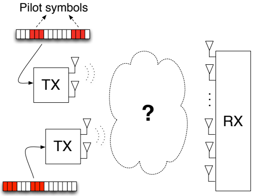

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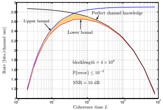
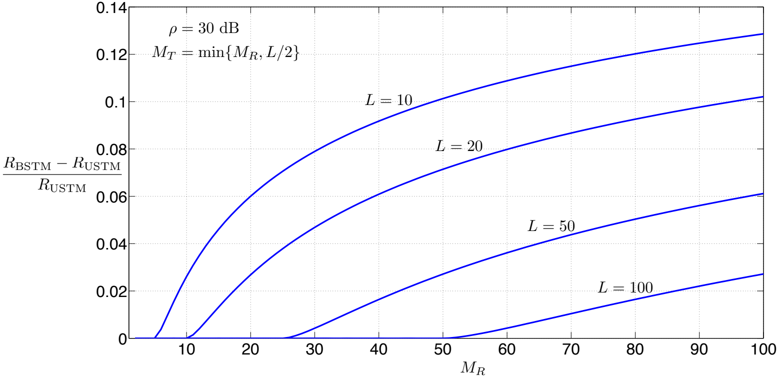
From asymptotia to finite blocklength



Backup Slides

Gain of BSTM over USTM for large-MIMO systems



Achievability for finite blocklength