Collaborative Interference Management:
When are the "Conventional Schemes" Optimal?

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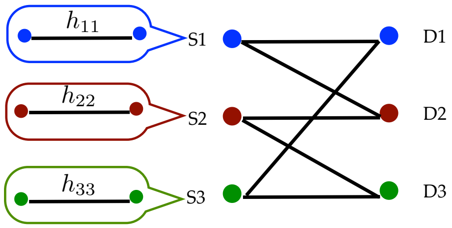
Interference Management Dichotomy

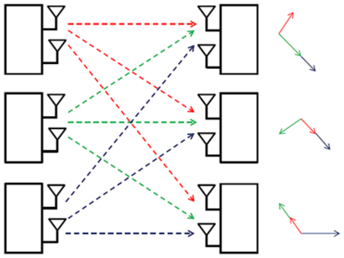
when to use each scheme?

conventional approaches



power control with
treat-interference-as-noise,
interference avoidance, etc.

modern approaches

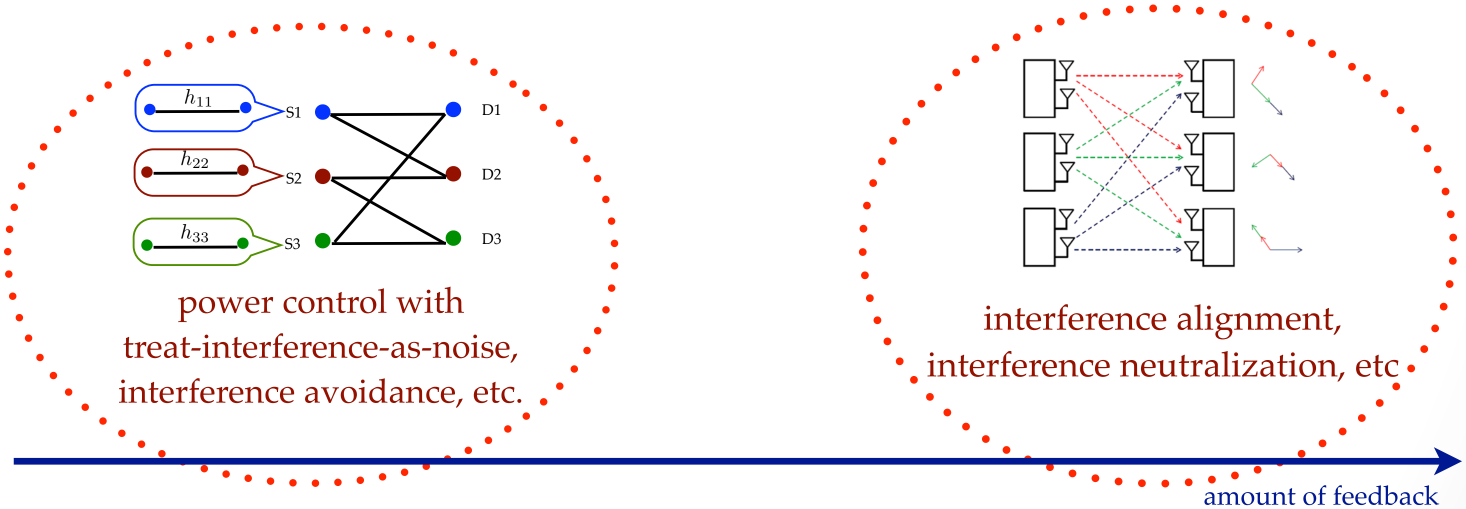


interference alignment,
interference neutralization, etc

amount of feedback
(to acquire CSI)

This talk ...

When is "power control + treat-interference-as-noise" optimal?



conventional approaches

power control with
treat-interference-as-noise,
interference avoidance, etc.

new approaches

interference alignment,
interference neutralization, etc

amount of feedback
(to acquire CSI)

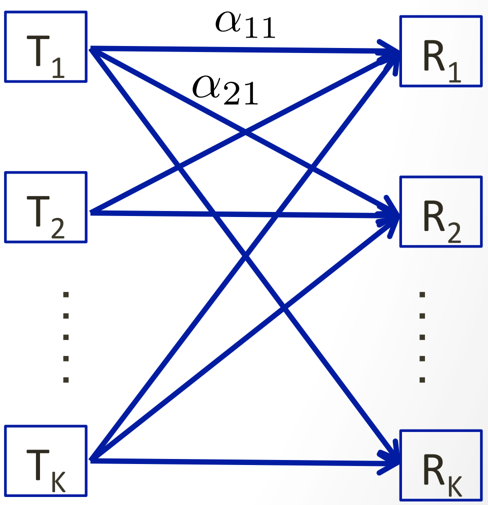
Setting

- K-user flat fading Gaussian interference channel
- Received signal at receiver j:

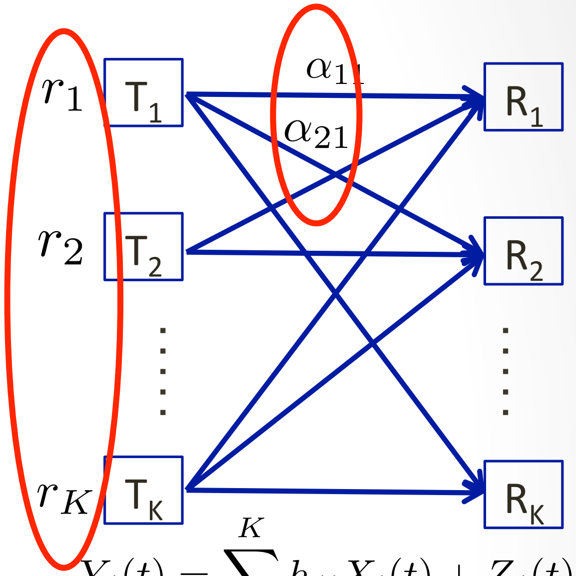
$$Y_j(t) = \sum_{i=1}^{K} h_{ji} \underbrace{X_i(t)}_{\text{power constraint 1}} + \underbrace{Z_j(t)}_{\mathcal{CN}(0,1)}$$

- Channel from T_i to R_j:

$$h_{ji} = \sqrt{P^{\underbrace{\alpha_{ji}}_{\text{channel strength (in dB, some base P>1)}}}} e^{\theta_{ji}}$$



Rate-Region of Treat-Interference-as-Noise (TIN)



$$Y_j(t) = \sum_{i=1}^{K} h_{ji} X_i(t) + Z_j(t)$$

- T_i uses power of P^{r_i} *(for some $r_i \leq 0$)*
- Signal-to-interference-plus-noise ratio (SINR) at R_i

$$\text{SINR}_i = \frac{P^{\alpha_{ii}} \times P^{r_i}}{1 + \sum_{j \neq i} P^{\alpha_{ij}} \times P^{r_j}}$$

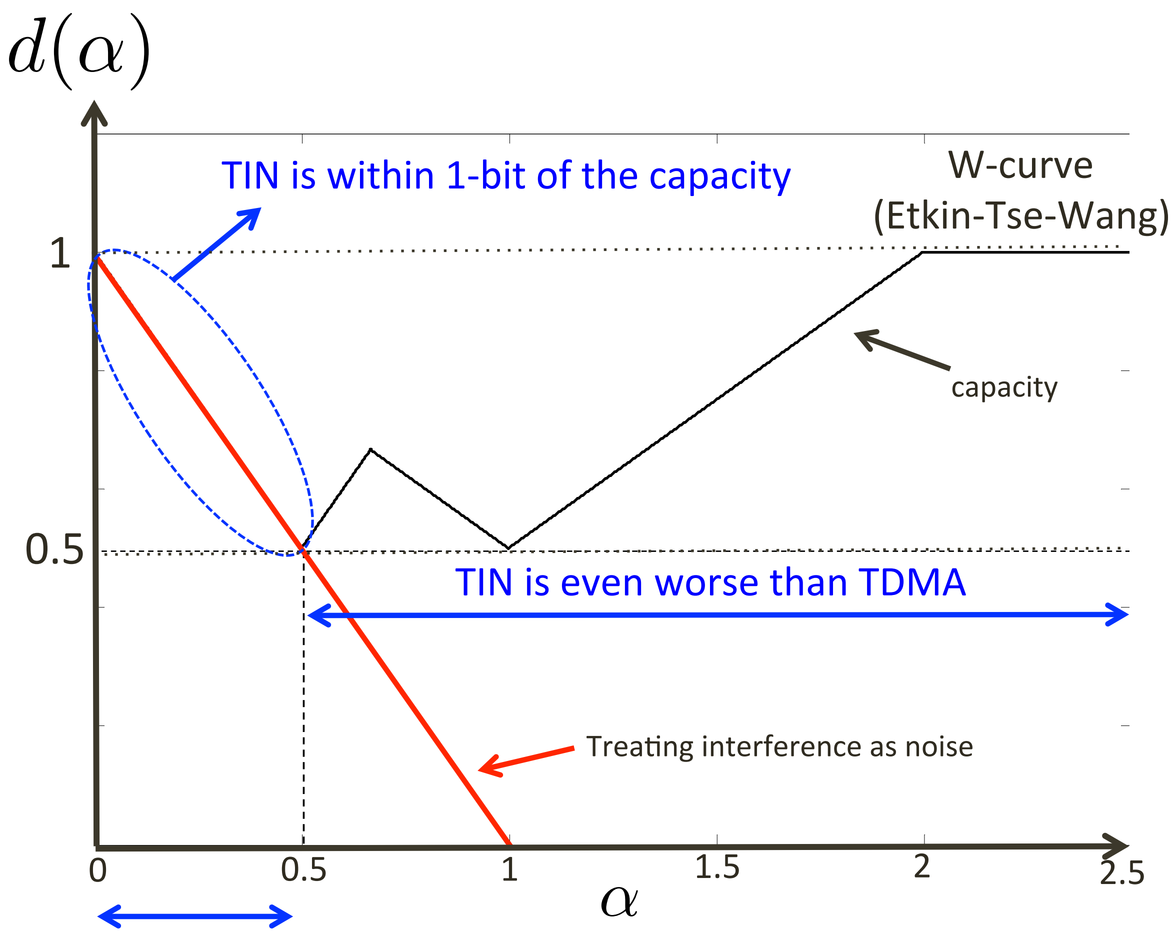
- Rate region of TIN

$$\mathcal{R}_{\text{TIN}} = \cup_{r_i \leq 0} \{(R_1, \ldots, R_K) : R_i = \log(1 + \text{SINR}_i)\}$$

- Generalized Degrees of Freedom (GDoF) region of TIN

$$\mathcal{D}_{\text{TIN}} = \lim_{P \to \infty} \frac{\mathcal{R}_{\text{TIN}}}{\log P}$$

What is known about the optimality of TIN?



TIN is within 1-bit of the capacity

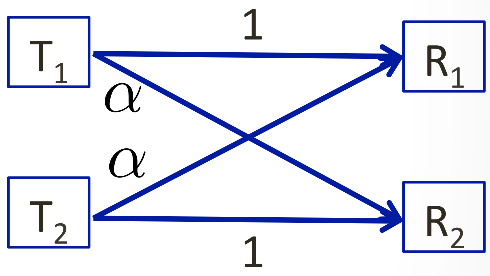
W-curve
(Etkin-Tse-Wang)

capacity

TIN is even worse than TDMA

Treating interference as noise

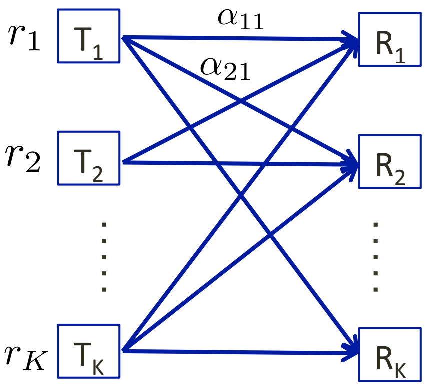
$$\mathrm{INR}_{dB} \leq \frac{1}{2} \times \mathrm{SNR}_{dB}$$



$$R_{\mathrm{TIN}} = \log(\cancel{1} + \frac{P}{\cancel{1} + P^{\alpha}})$$
$$\approx (1-\alpha)\log(P)$$

How about in general?

- Not much is known about its optimality (except for some symmetric and very low interference regimes)
- TIN region is hard to analyze analytically
  - Includes a (hard) optimization problem over r_i's (i.e., power allocation, e.g. Foschini-Milijanic 1993, Tan-Chiang-Srikant 2013)
  - Can be characterized through solving a sequence of GP's (Mahdavi et al 2008)
  - Non-explicit and non-convex region in general
- Very few general bounds on the capacity region of K-user IC



$$\mathcal{R}_{\text{TIN}} = \cup_{r_i \leq 0}\{(R_1, \dots, R_K) : R_i = \log(1 + \text{SINR}_i)\}$$

Main Result

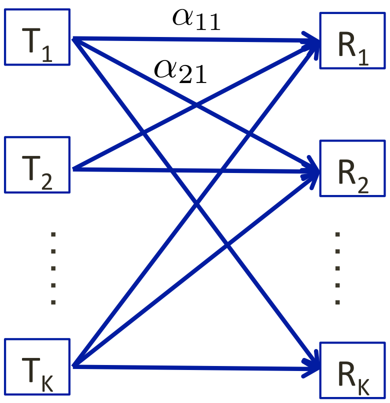
Theorem: In a *K*-user interference channel, if

$$\alpha_{ii} \geq \max_{j:j\neq i} \alpha_{ji} + \max_{k:k\neq i} \alpha_{ik}, \quad \forall i,j,k \in \{1,2,\dots,K\}$$

- TIN achieves the capacity region within a constant gap of $\log_2(3K)$ bits,
- TIN region is approximated by a polyhedron.

- In words, the condition is

"at each user, the desired channel strength is at least the sum of the strengths of the strongest interference from this user and the strongest interference to this user"



*C. Geng, N. Naderializadeh, A. S. Avestimehr, and S. A. Jafar "On the Optimality of Treating Interference as Noise", submitted to IT.

Some implications

Theorem: In a *K*-user interference channel, if

"at each user, the desired channel strength is at least the sum of the strengths of the strongest interference from this user and the strongest interference to this user"

- TIN achieves the capacity region within a constant gap of $\log_2(3K)$ bits,
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- ✧ Capacity approximation in a specific regime
- ✧ Underlying structure of the TIN rate-region
- ✧ A general condition for when acquiring additional CSI (e.g., phase) does not worth it

Key Challenges

Theorem: In a K-user interference channel, if

"at each user, the desired channel strength is at least the sum of the strengths of the strongest interference from this user and the strongest interference to this user"

- TIN achieves the capacity region within a constant gap of $\log_2(3K)$ bits,
- TIN region is approximated by a polyhedron.

- Explicit characterization of the TIN rate-region.

- Matching outerbounds.

Step 1: polyhedral relaxation of TIN

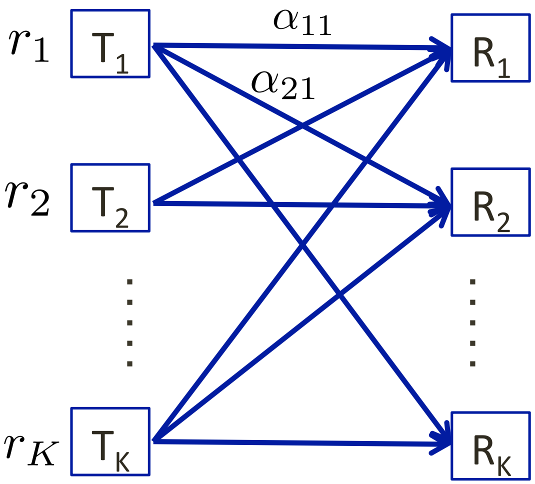
- Rate of each user (for a given power allocation):

$$d_i = \frac{R_i}{\log P} \approx \alpha_{ii} + r_i - \max_{j\neq i}(\alpha_{ij} + r_j)^+$$

- Polyhedral relaxation of TIN:

a polyhedron with K faces

$$\bigcup_{r_i \le 0} \left\{ \begin{array}{ll} d_i \le \alpha_{ii} + r_i & \\ d_i \le (\alpha_{ii}+r_i) - (\alpha_{ij}+r_j), & j \neq i \end{array} \right.$$



GP relaxation

$$R_i = \log\left(\cancel{1} + \frac{P^{\alpha_{ii}+r_i}}{1 + \sum_{j\neq i} P^{\alpha_{ij}+r_j}}\right)$$

$$\approx P^{\max_{j\neq i}(\alpha_{ij}+r_j)^+}$$

Step 2: Relation to Potential Functions

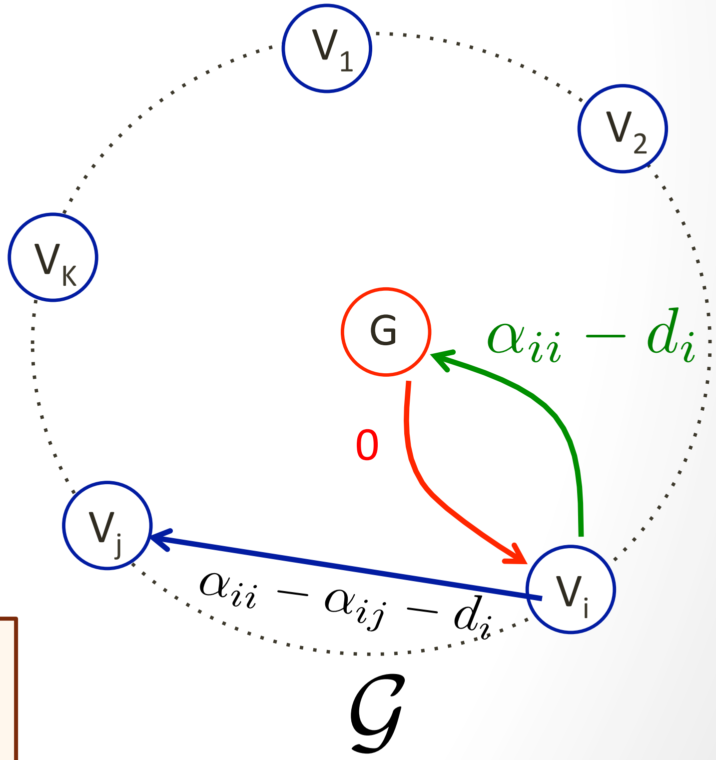
- Polyhedral TIN:

$$\bigcup_{r_i\le 0}\begin{cases} d_i\le\alpha_{ii}+r_i \\ d_i\le(\alpha_{ii}+r_i)-(\alpha_{ij}+r_j), \quad j\ne i\end{cases}$$

  - It is the set of all (d_1,…,d_K) for which there exists (r_1,…,r_K) satisfying

$$\begin{aligned} r_i &\le 0 \\ -r_i &\le \alpha_{ii}-d_i \\ r_j-r_i &\le \alpha_{ii}-\alpha_{ij}-d_i \end{aligned}$$

  - It is the set of all (d_1,…,d_K) for which there exists a potential function on



$r : V \to \mathrm{R}$, is a potential function on $G=(V,E,w)$, if

$$r(j)-r(i)\le w(i,j), \quad \forall i,j\in V$$

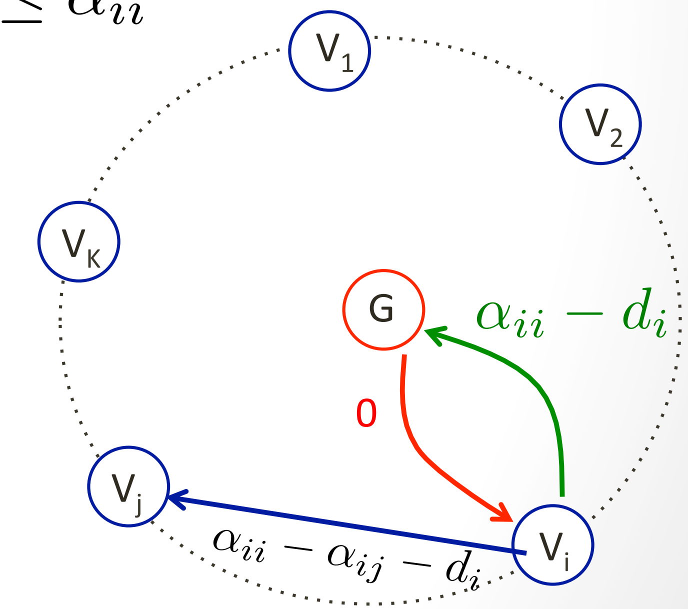
Step 3: Potential Theorem

- **Theorem [Hoffman]:** There exists a potential function for a directed graph *D* if and only if all directed loops have non-negative weights.

- Therefore, $(d_1,\ldots,d_K)$ is in polyhedral TIN if and only if

  - $(\mathrm{G}, \mathrm{V}_i, \mathrm{G}) : \alpha_{ii} - d_i \geq 0 \Rightarrow d_i \leq \alpha_{ii}$
  - $(\mathrm{V}_{i_0}, \mathrm{V}_{i_1}, \ldots, \mathrm{V}_{i_m} = \mathrm{V}_{i_0}) :$

$$\sum_{\ell=0}^{m-1} d_{i_\ell} \leq \sum_{\ell=0}^{m-1} (\alpha_{i_\ell i_\ell} - \alpha_{i_\ell i_{\ell+1}})$$



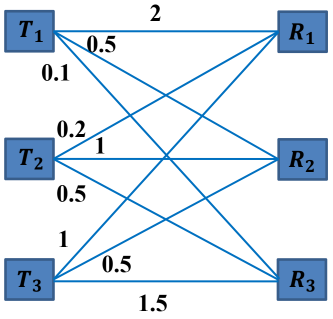
Recap

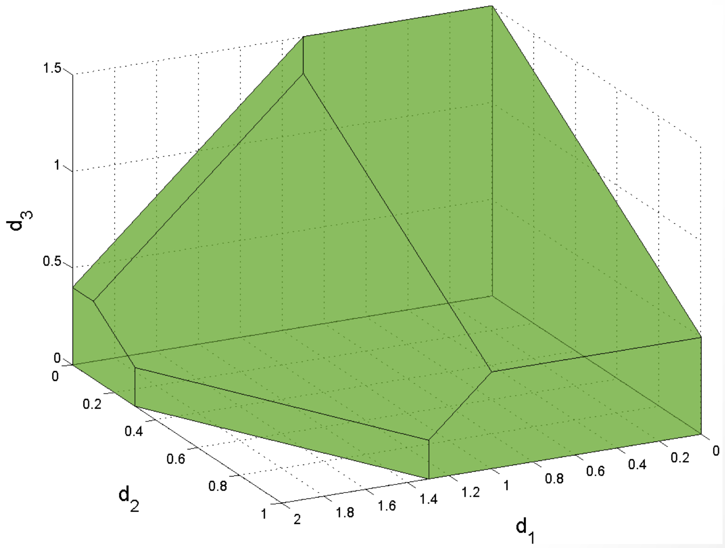
- Polyhedral TIN: $\cup_{r_i \leq 0} \begin{cases} d_i \leq \alpha_{ii} + r_i \\ d_i \leq (\alpha_{ii} + r_i) - (\alpha_{ij} + r_j), \quad j \neq i \end{cases}$

(potential thm) $|||$

r_i's are gone ☺

$$(d_1, \ldots, d_K) \text{ s.t. } \begin{cases} 0 \leq d_i \leq \alpha_{ii} \\ \sum_{\ell=0}^{m-1} d_{i_\ell} \leq \sum_{\ell=0}^{m-1} (\alpha_{i_\ell i_\ell} - \alpha_{i_\ell i_{\ell+1}}) \end{cases}$$





Step 4: Matching Outerbounds

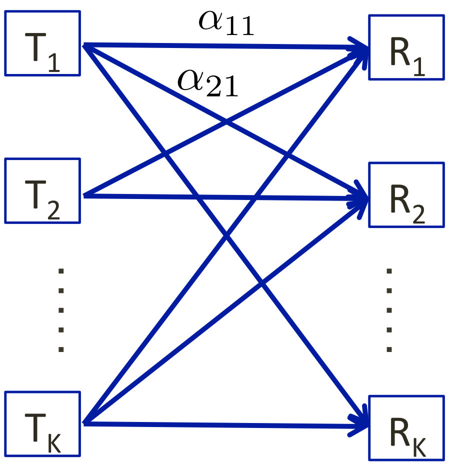
- Lemma: Consider a K user interference channel, such that

$$\alpha_{ii} \geq \max_{j:j\neq i} \alpha_{ji} + \max_{k:k\neq i} \alpha_{ik}, \quad \forall i,j,k \in \{1,2,\ldots,K\}$$

then any (d_1,…,d_K) in the GDoF region satisfies

$$0 \leq d_i \leq \alpha_{ii}$$

$$\sum_{\ell=0}^{m-1} d_{i_\ell} \leq \sum_{\ell=0}^{m-1} (\alpha_{i_\ell i_\ell} - \alpha_{i_\ell i_{\ell+1}})$$



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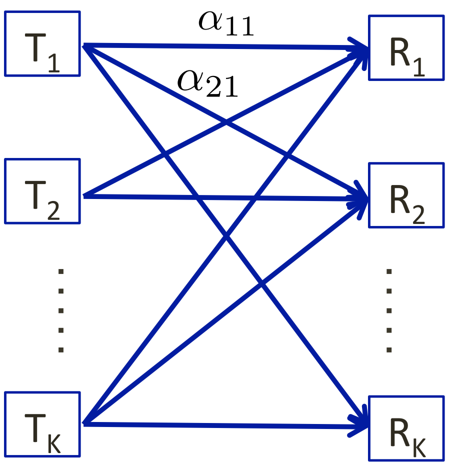

$$(d_1,\ldots,d_K) \text{ s.t. } \begin{cases} 0\leq d_i\leq\alpha_{ii} \\ \sum_{\ell=0}^{m-1} d_{i_\ell} \leq \sum_{\ell=0}^{m-1}(\alpha_{i_\ell i_\ell}-\alpha_{i_\ell i_{\ell+1}}) \end{cases}$$

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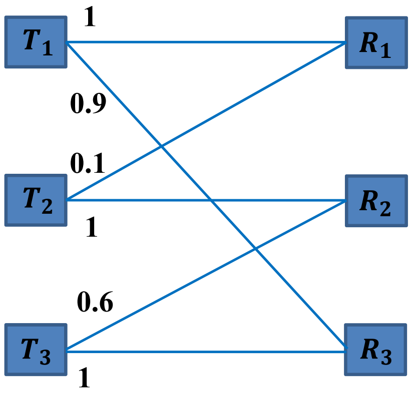
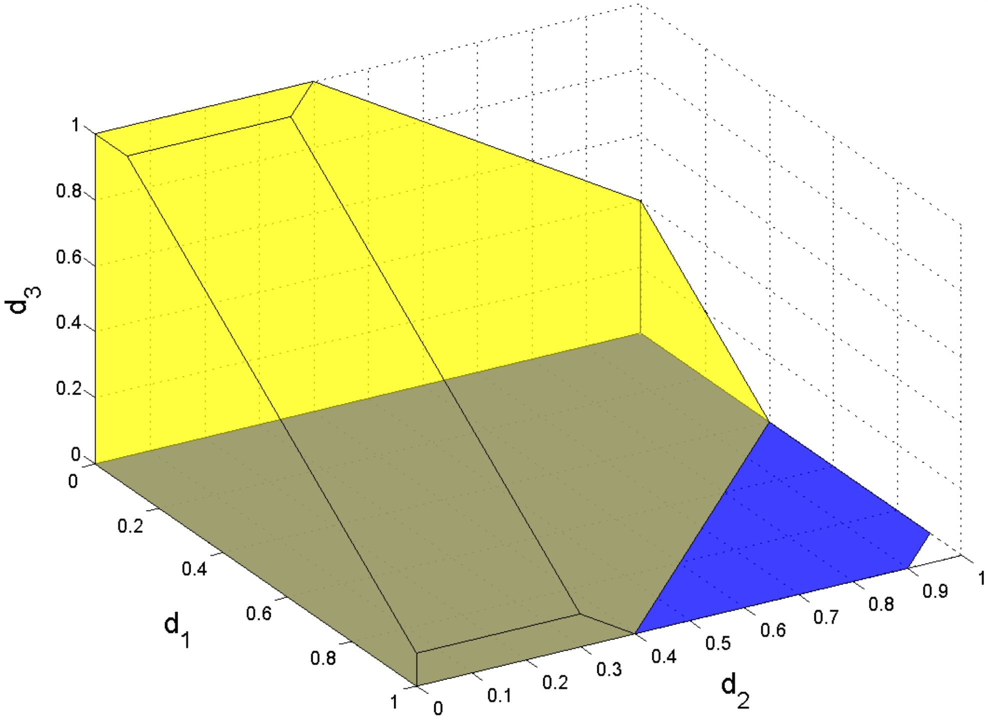
Rate-Region of TIN in General

- **Theorem:** In a *K*-user interference channel, the rate region achievable by TIN is within $\log_2(3K)$ bits of

$$\mathcal{P}^* = \bigcup_{\mathcal{S}\subseteq\{1,\dots,K\}} \mathcal{P}_{\mathcal{S}}$$

where P_S is the polyhedral TIN region when the users in *S* are silent.





Necessary and Sufficient Conditions

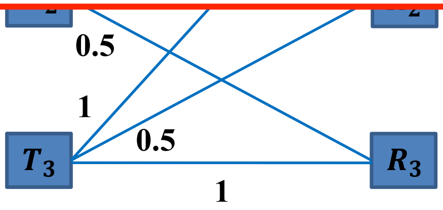
- Is the condition both necessary and sufficient for optimality of TIN (to within a constant gap)?

$$\alpha_{ii} \geq \max_{j:j\neq i} \alpha_{ji} + \max_{k:k\neq i} \alpha_{ik}, \quad \forall i,j,k \in \{1,2,\ldots,K\}$$

Conjecture: In a *K*-user interference channel, TIN is constant-gap optimal if and only if

$$\alpha_{ii} \geq \max_{j:j\neq i}\{\alpha_{ji}\} + \max_{k:k\neq i}\{\alpha_{ik}\}, \quad \forall i,j,k \in \{1,2,...K\}$$

except for a set of measure-zero gains.

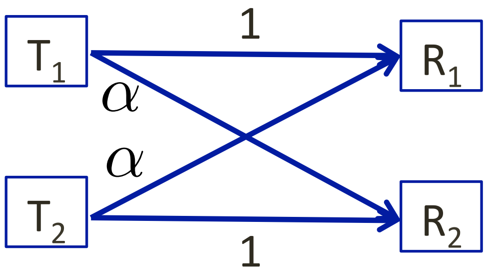


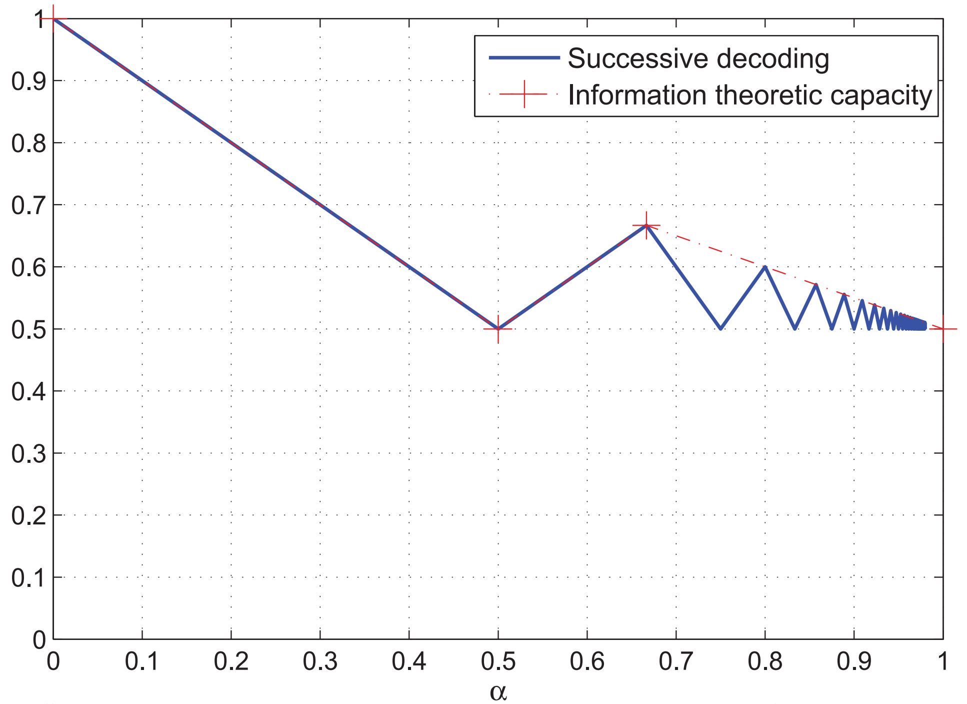
all-phases zero

➢ However, once the phases are perturbed, TIN isn't constant-gap optimal anymore

Successive Interference Cancellation

- Rate region can be increased, using
  - Superposition coding
  - Successive interference cancellation
  - Treat-interference-as-noise
- When would that be optimal?





*Y. Zhao, C. Tan†, S. Avestimehr‡, S. Diggavi and G. Pottie “On the Sum-Capacity with Successive Decoding in Interference Channels”, IT 2012..

Concluding remarks

- Established general conditions for (approximate) optimality of power control + treat-interference-as-noise
  - A general condition for when acquiring additional CSI does not worth it
  - Underlying structure of the TIN rate-region
- Network-level analysis of the capacity gains?
- How much can one go beyond TIN when CSI is only the channel gain, or channel-gain + some local phase?

Questions?