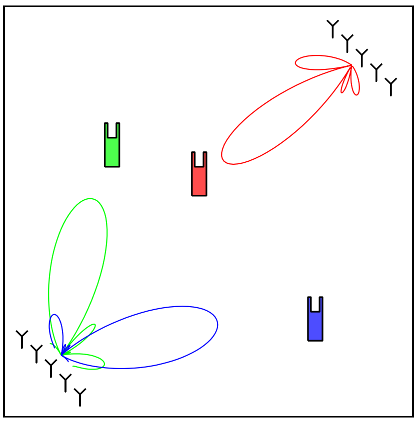
MU-MIMO System Level Utility Optimization, Revisited

Mats Bengtsson, Emil Björnson, Rasmus Brandt
Signal Processing, KTH

The Problem

- Multiple users
- Multiple cells
- Multiple antennas (MISO or MIMO)
- Downlink transmission
- Utility optimization



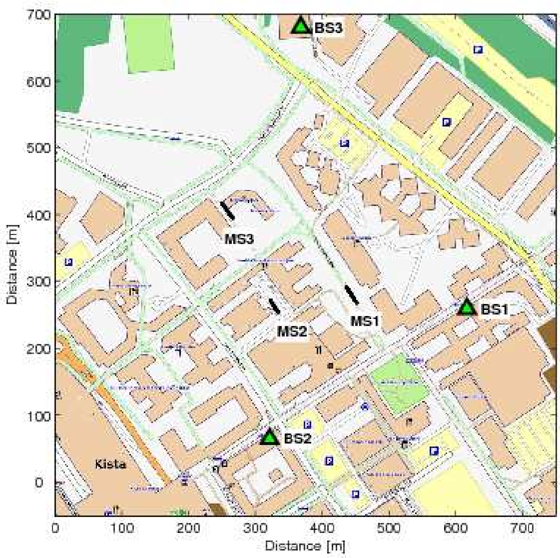
Problem Features

- Interference Channel
- NP-difficult
- Recent results: Interference Alignment
- Many algorithms
  - Alternating optimization
  - Decomposition
  - . . .

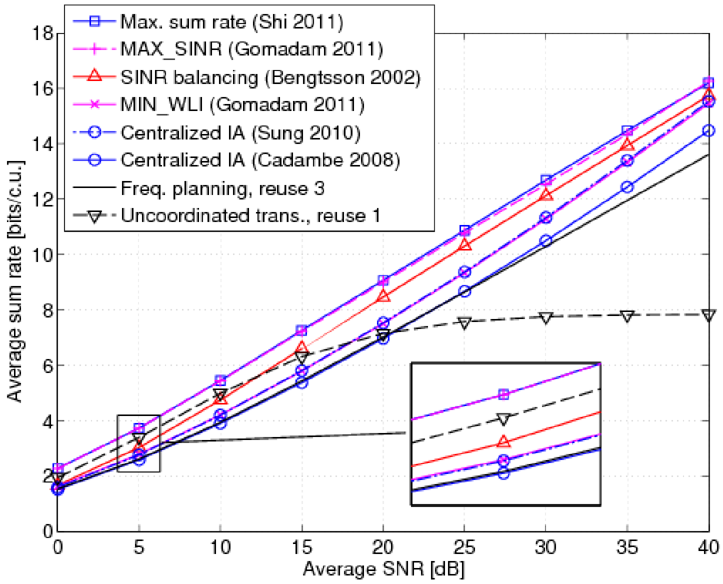
Motivating Example, Interference Alignment in Frequency

- 3 subcarriers $\Longrightarrow$ diagonal "MIMO" channels
- Measurements from Ericsson.
- 3 users, 4 streams

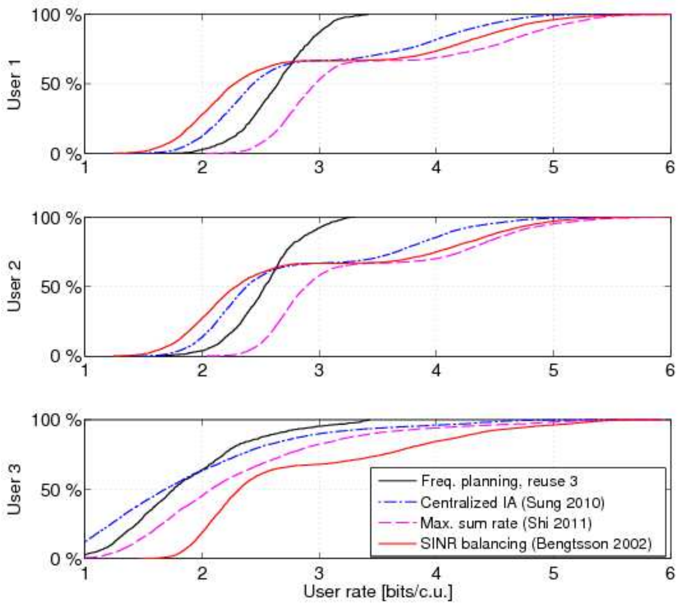
Brandt, Asplund & Bengtsson, "Interference Alignment in Frequency – a Measurement Based Performance Analysis", Proc. IWSSIP 2012.



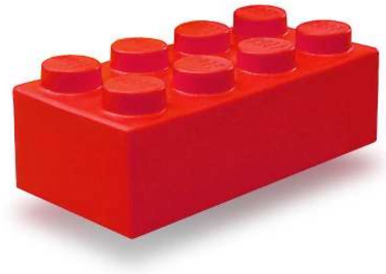
Example, Results



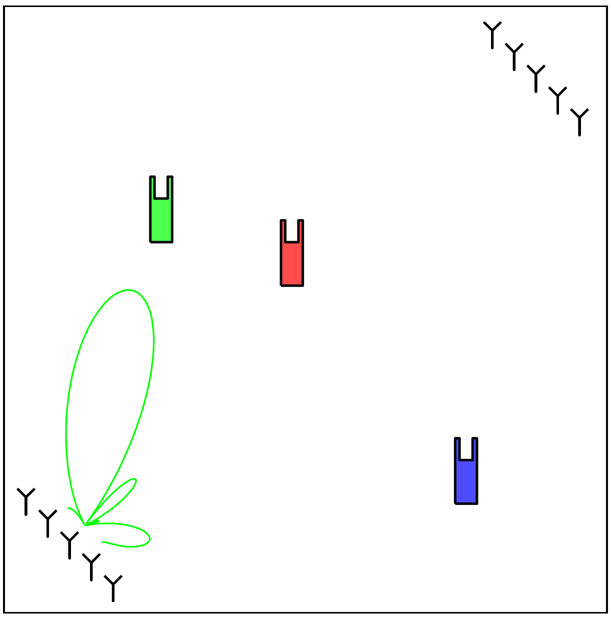
Example, Results



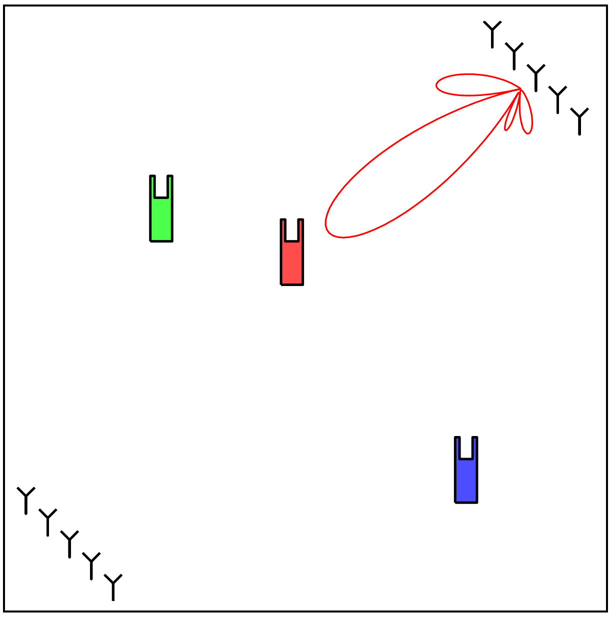
Strategy – Decomposition



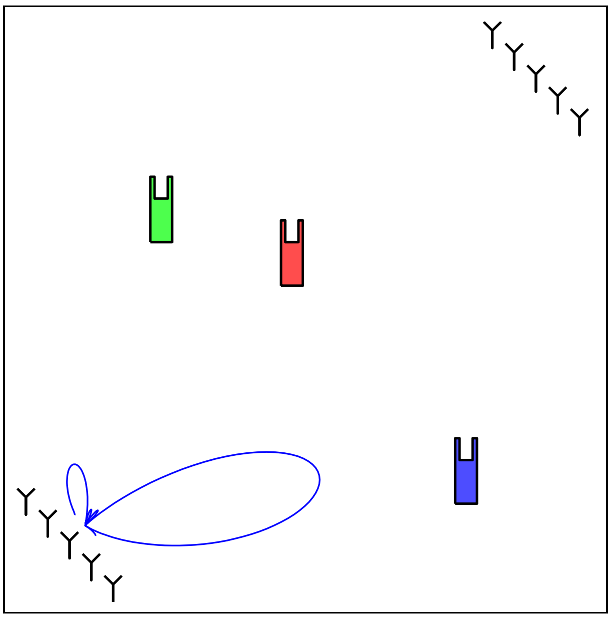
Subproblems



Subproblems

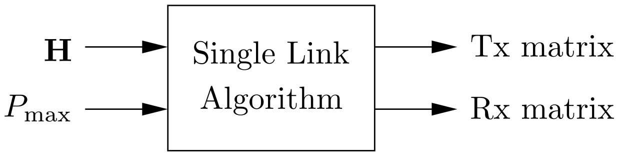


Subproblems



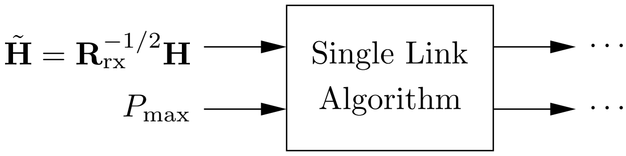
Interference Suppression using Prewhitening

Assume given: Algorithms for a single link in white noise. E.g. optimize MMSE or SINR or mutual information or BER or ...



Interference suppression at the receivers:

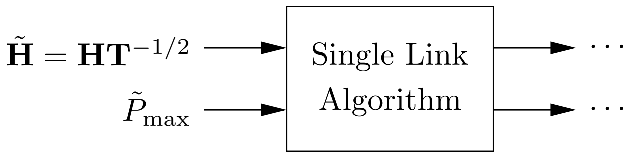
Noise prewhitening at the receivers (well-known).



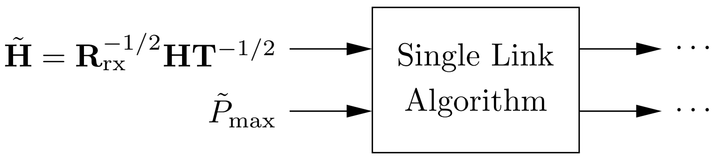
Interference Suppression, cont.

Interference suppression at the transmitters:

"Noise prewhitening" at the transmitters.



Combine:



Iterate! Circular dependence,

$\mathbf{R}_{\text{rx}}$ and $\mathbf{T}$ $\Longleftrightarrow$ Tx and Rx parameters.

Solve iteratively for each link! Hope for convergence!

Historical Review

2009 Zakhour&Gesbert, "Coordination on the MISO Interference Channel Using the Virtual SINR Framework"

Virtual SINR

2009 Lee, Je, Shin & Lee, "A Novel Uplink MIMO Transmission Scheme in a Multicell Environment"

Signal to Generated Interference plus Noise Ratio, SGINR

KTH Electrical Engineering

2007 Sadek, Tarighat & Sayed, "A Leakage-Based Precoding Scheme for Downlink Multi-User MIMO Channels"

Signal-to-Leakage-and-Noise Ratio, SLNR

2005 Joham, Utschick & Nossek, "Linear Transmit Processing in MIMO Communication Systems"

Transmit Wiener Filter

Keeping Digging . . .

1998 Goldberg & Fonollosa, "Downlink Downlink Beamforming for Spatially Distributed Sources in Cellular Mobile Communications"

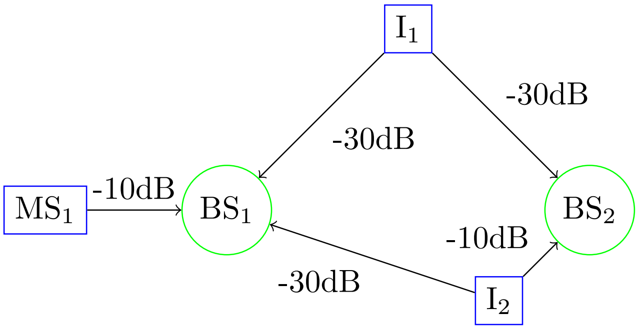
Minimum Average Interference Power Criterion

KTH Electrical Engineering

1996 Gerlach & Paulraj, "Base Station Transmitting Antenna Arrays for Multipath Environments"

1995 Zetterberg & Ottersten, "The Spectrum Efficiency of a Base Station Antenna Array for Spatially Selective Transmission"

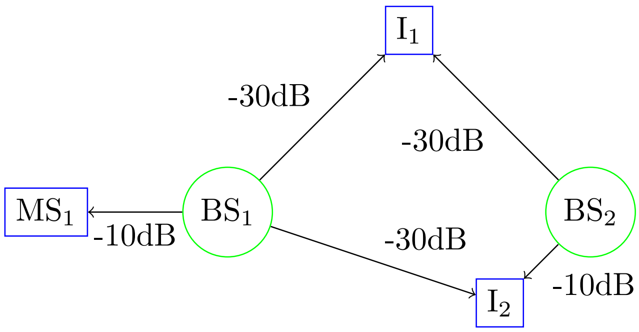
Why Uplink ≠ Downlink



Uplink:

Receiving beamformer at BS_1 should suppress I_1 and I_2 equally much!

Why Uplink ≠ Downlink, cont.



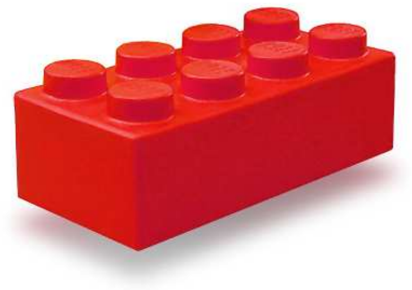
Downlink:

Transmitting beamformer at BS_1 should suppress 20dB more towards I_1 than towards I_2!

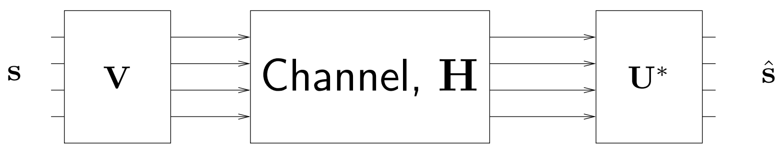
My LEGO Brick

Palomar, Cioffi, Lagunas, “Joint Tx-Rx Beamforming Design for Multicarrier MIMO Channels: A Unified Framework for Convex Optimization”, IEEE Trans SP, 2003.

IEEE SP Young Author Best Paper Award 2004!



Single Link Model



$$\hat{\mathbf{s}} = \mathbf{U}^*(\mathbf{H}\mathbf{V}\mathbf{s} + \mathbf{n})$$

$\mathbf{V}$: Linear transmit processing

$\mathbf{U}$: Linear receive processing

Rx strategy: MMSE “always” optimal

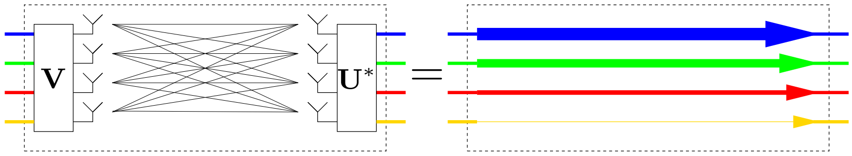
Resulting MMSE: $\mathbf{E}(\mathbf{V}) = \left(\mathbf{I} + \mathbf{V}^H\mathbf{H}^H\mathbf{R}_{\mathrm{IN}}^{-1}\mathbf{H}\mathbf{V}\right)^{-1}$

Tx strategy: $\min f\left(\mathrm{diag}\{\mathbf{E}(\mathbf{V})\}\right)$

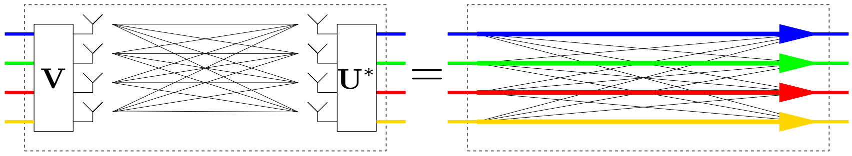
Palomar, Unified Analysis

Two main transmit solutions!

Shur-Concave Criteria: MMSE, Max mutual information, mean SINR across subchannels, ...



Shur-Convex Criteria: Max MSE across subchannels, mean&max BER across subchannels, ...



System Level Problem Formulation

Link level utility: $u(\mathbf{V}_k; \mathbf{H}_{kk}, \mathbf{R}_{\mathrm{IN}_k})$

Interference+noise cov.: $\mathbf{R}_{\mathrm{IN}_k} = \sum_{n \neq k} \mathbf{H}_{nk} \mathbf{V}_n \mathbf{V}_n^H \mathbf{H}_{nk}^H + \sigma_k^2 \mathbf{I}$

System level utility: $\min_{\mathbf{V}_k} \sum_k u(\mathbf{V}_k; \mathbf{H}_{kk}, \mathbf{R}_{\mathrm{IN}_k})$

Power constraints per transmitter E.g. $\mathrm{Tr}[\mathbf{V}_k \mathbf{V}_k^H] \leq P_k$ and/or per antenna, ...

Decomposition Trick

$$u(\mathbf{V};\mathbf{H},\mathbf{R}_{\text{IN}}) = \max_{\boldsymbol{\Psi}\preceq\mathbf{R}_{\text{IN}}} u(\mathbf{V};\mathbf{H},\boldsymbol{\Psi})$$

$$\Longrightarrow$$

$$\begin{aligned}\min_{\{\mathbf{V}_k\}}\max_{\{\boldsymbol{\Phi}_k\}} & \; u(\mathbf{V}_k;\mathbf{H}_{kk},\boldsymbol{\Psi}_k)\\ \text{s.t. } & \; \boldsymbol{\Psi}_k \preceq \mathbf{H}_{nk}\mathbf{V}_n\mathbf{V}_n^H\mathbf{H}_{nk}^H + \sigma_k^2\mathbf{I}\\ & \; \text{Power constr.}\{\mathbf{V}_k\}\end{aligned}$$

Lagrangian for $\boldsymbol{\Psi}_k \preceq \mathbf{R}_{\text{IN}_k}$

$$\sum_k u(\mathbf{V}_k;\mathbf{H}_{kk},\boldsymbol{\Psi}_k)+\sum_k \operatorname{Tr}\left[\boldsymbol{\Lambda}_k\sum_{n\neq k}\mathbf{H}_{nk}\mathbf{V}_n\mathbf{V}_n^H\mathbf{H}_{nk}^H\right]+\sum_k\sigma_k^2\operatorname{Tr}[\boldsymbol{\Lambda}_k]-\sum_k\operatorname{Tr}[\boldsymbol{\Lambda}_k\boldsymbol{\Psi}_k]$$

$$=\sum_m\left(u(\mathbf{V}_m;\mathbf{H}_{mm},\boldsymbol{\Psi}_m)-\operatorname{Tr}[\boldsymbol{\Lambda}_m\boldsymbol{\Psi}_m]+\operatorname{Tr}\Big[\mathbf{V}_m\mathbf{V}_m^H\sum_{l\neq m}\mathbf{H}_{ml}^H\boldsymbol{\Lambda}_l\mathbf{H}_{ml}\Big]\right)+\sum_m\sigma_m^2\operatorname{Tr}[\boldsymbol{\Lambda}_m]$$

Decomposition into subproblems

$$\min_{\mathbf{V}}\max_{\boldsymbol{\Phi}_k} u(\mathbf{V}_m;\mathbf{H}_{mm},\boldsymbol{\Psi}_m)-\operatorname{Tr}[\boldsymbol{\Lambda}_m\boldsymbol{\Psi}_m]+\operatorname{Tr}\Big[\mathbf{V}_m\mathbf{V}_m^H\underbrace{\sum_{l\neq m}\mathbf{H}_{ml}^H\boldsymbol{\Lambda}_l\mathbf{H}_{ml}}_{\triangleq\mathbf{T}}\Big]$$

$$\text{s.t. Power constr.}\{\mathbf{V}_k\}$$

Decomposition, cont.

Single link dual:

$$\tilde{u}(\mathbf{\Lambda}, \mathbf{T}; \mathbf{H}) = \min_{\mathbf{V}} \max_{\mathbf{\Phi}} u(\mathbf{V}; \mathbf{H}, \mathbf{\Psi}) - \mathrm{Tr}[\mathbf{\Lambda}\mathbf{\Psi}] + \mathrm{Tr}\left[\mathbf{V}\mathbf{V}^H\mathbf{T}\right]$$
$$\text{s.t. Power constr.}\{\mathbf{V}\}$$

Lagrange including power constr.:

$$u(\mathbf{V}; \mathbf{H}, \mathbf{\Psi}) - \mathrm{Tr}[\mathbf{\Lambda}\mathbf{\Psi}] + \mathrm{Tr}\left[\mathbf{V}\mathbf{V}^H(\mathbf{T}+\mathbf{Q})\right]$$

$\mathbf{T}+\mathbf{Q}$: Lagrangian of another power constraint

Ex. total power/BS: $\mathbf{Q}_k = q_k\mathbf{I}$

System level dual:

$$\min_{\{\mathbf{\Lambda}_k \succeq 0\}} \tilde{u}\left(\mathbf{\Lambda}_k, \sum_{l\neq k} \mathbf{H}_{kl}^H \mathbf{\Lambda}_l \mathbf{H}_{kl}; \mathbf{H}_{kk}\right)$$

MMSE Based Utility

$$\min_{\mathbf{V}} \max_{\mathbf{\Phi}} f\left(\mathrm{diag}\left[\left(\mathbf{I} + \mathbf{V}^H\mathbf{H}^H\mathbf{\Psi}^{-1}\mathbf{H}\mathbf{V}\right)^{-1}\right]\right) - \mathrm{Tr}[\mathbf{\Lambda}\mathbf{\Psi}] + \mathrm{Tr}\left[\mathbf{V}\mathbf{V}^H(\mathbf{T}+\mathbf{Q})\right]$$

Prewhitening:

$$\tilde{\mathbf{V}} = (\mathbf{T}+\mathbf{Q})^{1/2}\mathbf{V}$$
$$\tilde{\mathbf{\Psi}} = \mathbf{\Lambda}^{1/2}\mathbf{\Psi}\mathbf{\Lambda}^{H/2}$$
$$\tilde{\mathbf{H}} = \mathbf{\Lambda}^{1/2}\mathbf{H}(\mathbf{T}+\mathbf{Q})^{-1/2}$$

Canonical single link problem:

$$\min_{\tilde{\mathbf{V}}} \max_{\tilde{\mathbf{\Psi}}} f\left(\mathrm{diag}\left[\left(\mathbf{I} + \tilde{\mathbf{V}}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{\Psi}}^{-1}\tilde{\mathbf{H}}\tilde{\mathbf{V}}\right)^{-1}\right]\right) - \mathrm{Tr}[\tilde{\mathbf{\Psi}}] + \mathrm{Tr}\left[\tilde{\mathbf{V}}\tilde{\mathbf{V}}^H\right]$$
$$= \min_{\tilde{\mathbf{V}}} \max_{\tilde{\mathbf{\Psi}}} u(\tilde{\mathbf{V}}; \tilde{\mathbf{H}}, \tilde{\mathbf{\Psi}}) - \mathrm{Tr}[\tilde{\mathbf{\Psi}}] + \mathrm{Tr}[\tilde{\mathbf{V}}\tilde{\mathbf{V}}^H]$$

Palomar, Revisited

Modifications needed!

Notation:

$$\tilde{\rho}_k = \sigma_k[\tilde{\mathbf{H}}]^2$$
$$\tilde{\gamma}_k = \sigma_k[\tilde{\mathbf{V}}_k]^2$$
$$\tilde{\psi}_k = \lambda_k[\tilde{\mathbf{\Psi}}]$$

Schur-concave criteria: Diagonalization optimal!

$$u(\tilde{\mathbf{V}};\tilde{\mathbf{H}},\tilde{\mathbf{\Psi}}) - \mathrm{Tr}[\tilde{\mathbf{\Psi}}] + \mathrm{Tr}[\tilde{\mathbf{V}}\tilde{\mathbf{V}}^H] = f\left(\left[1+\frac{\tilde{\rho}_k\tilde{\gamma}_k}{\tilde{\psi}_k}\right]\right) - \sum_k \tilde{\psi}_k + \sum_k \tilde{\gamma}_k$$

Generic solution: $\gamma_k = \psi_k$!!!

$$\min_{\{\tilde{\gamma}_k\}} \max_{\{\tilde{\psi}_k\}} f\left(\left[1+\frac{\tilde{\rho}_k\tilde{\gamma}_k}{\tilde{\psi}_k}\right]\right) - \sum_k \tilde{\psi}_k + \sum_k \tilde{\gamma}_k = f([1+\tilde{\rho}_k]) \text{ !!!}$$

Palomar, Revisited, cont.

Schur-convex criteria: Diagonalization + FFT matrix optimal!
Solution independent on $f(\cdot)$!

$$u(\tilde{\mathbf{V}};\tilde{\mathbf{H}},\tilde{\mathbf{\Psi}}) - \mathrm{Tr}[\tilde{\mathbf{\Psi}}] + \mathrm{Tr}[\tilde{\mathbf{V}}\tilde{\mathbf{V}}^H] = f\left(\frac{1}{K}\sum_k 1+\frac{\tilde{\rho}_k\tilde{\gamma}_k}{\tilde{\psi}_k}\right) - \sum_k \tilde{\psi}_k + \sum_k \tilde{\gamma}_k$$

Generic solution: $\gamma_k = \psi_k$!!!

$$\min_{\{\tilde{\gamma}_k\}} \max_{\{\tilde{\psi}_k\}} f\left(\left[1+\frac{\tilde{\rho}_k\tilde{\gamma}_k}{\tilde{\psi}_k}\right]\right) - \sum_k \tilde{\psi}_k + \sum_k \tilde{\gamma}_k = f\left(1+\frac{1}{K}\sum_k \tilde{\rho}_k\right) \text{ !!!}$$

Resulting System Level Dual

Observation:

$$1+\tilde{\rho}_k = \lambda_k \mathbf{I} + \tilde{\mathbf{H}}\tilde{\mathbf{H}}^H = \lambda_k \{\mathbf{I} + \mathbf{\Lambda}^{1/2}\mathbf{H}(\mathbf{T}+\mathbf{Q})^{-1}\mathbf{H}^H\mathbf{\Lambda}^{H/2}\}$$

System level dual:

$$\min_{\{\mathbf{\Lambda}_k \succeq 0\}} \sum_k f\left(\lambda_m \left\{\mathbf{I} + \mathbf{\Lambda}_k^{1/2}\mathbf{H}_{kk}(\sum_{l\neq k}\mathbf{\Lambda}_l\mathbf{H}_{kl} + \mathbf{Q}_k)^{-1}\mathbf{H}_{kl}^H\mathbf{\Lambda}_l^{H/2}\right\}\right)$$

Compare uplink–downlink duality: MMSE{downlink} = MMSE{virtual uplink}, $\mathbf{\Lambda}_k$=virtual uplink transmit covariance

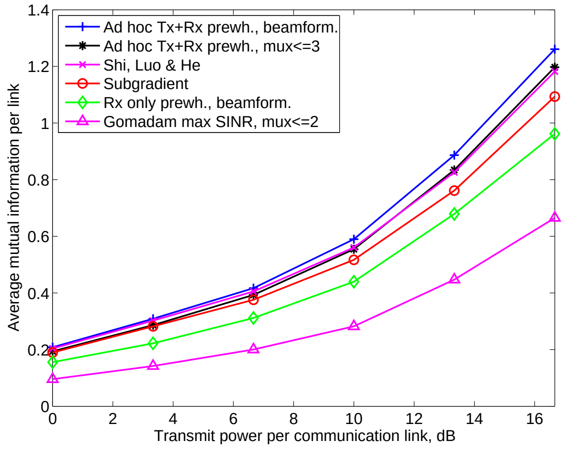
Algorithms

Several alternatives!

- Subgradient search over $\mathbf{\Lambda}$.
- Ad-hoc choices of $\mathbf{\Lambda}$, $\mathbf{T}$:
  - Use downlink equalizer as uplink precoder. Use $\mathbf{T}$ = receive covariance in true uplink. Iterate!
  - Signal to Leakage and Noise
  - ...

Numerical Example, Prewhitening

- 4 access points
- 8 user terminals, random position
- Path loss + shadow fading
- Each user served by strongest AP
- 4×4 MIMO



Extensions

- Coordinated Multipoint
- OFDMA
- Other system level utilities
  - Weighted sum utility
  - Non-linear functions of u_k?
- Robustified versions
- Cognitive radio
- Algorithms
- ...

Conclusions

- Yet another derivation of uplink–downlink duality!
- Structural result — Tx+Rx prewhitening optimal!
- Based on Palomar's single-link solutions,
  Shur-convex/Schur-concave utility functions!
- Algorithmic ideas!