Optimization in Communication Networks

Session Organizer:

Wei Yu
Dept. of Electrical and Computer Engineering
University of Toronto

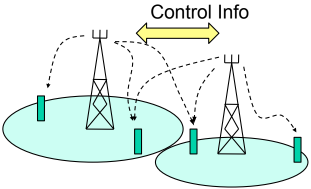
May 15, 2012

Talks

- 2:15-2:30 Overview
- 2:30-3:00 **Stephen Hanly** (Macquarie University, Australia)
  - New Insights in Coordinated Beamforming for Cellular Systems via Large-System Analysis
- 3:00-3:30 **Mats Bengtsson** (KTH)
  - System-Level Utility Optimization, Revisted
- 3:30-3:45 (Coffee Break)
- 3:45-4:15 **Zhi-Quan Luo** (University of Minnesota)
  - Base-Station Assignment and Transciever Design for Heterogeneous Networks
- 4:15-4:45 **Rui Zhang** (National University of Singapore)
  - Cooperative Beamforming for MISO Interference Channel: Achievable Rates and Distributed Algorithms

Wireless Multicell Network

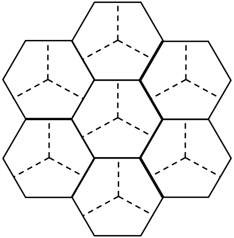
- Inter-cell interference is the fundamental limiting factor
- Coordination at the signal level: Network MIMO
  - Base-stations form an giant antenna array
  - Joint signal processing
- Coordination of signalling strategies:
  - Coordinating power spectrum
  - Coordinating beamforming
  - Coordinating scheduling



- Network optimization plays a key role.

Problem Formulation

- Consider a cooperative system at signaling-strategy level.
- System setup:
  - Multi-cell, sectorized, MIMO-OFDM
  - Multiple antennas at both BS/MS
  - Users occupy orthogonal dimensions
- Optimization objective:
  - Network utility maximization



- Optimization variables:
  - Scheduling: Which user in each dimension: $k = f(l, s, b, n)$.
  - Beamforming: What are the transmit/receive BF: (u_{lsb}^n, v_{lsk}^n).
  - Power control: What are the power levels for each beam: P_{lsb}^n.

Mathematical Formulation

- For lth cell, sth sector, bth beam, kth user, nth frequency

$$\mathrm{SINR}^n_{lsbk} = \frac{P^n_{lsb}|(u^n_{lsk})^T H^n_{ls,lsk} v^n_{lsb}|^2}{\Gamma(\sigma^2 + \sum_{(j,t,c)\neq(l,s,b)} P^n_{jtc}|(u^n_{lsk})^T H^n_{jt,lsk} v^n_{jtc}|^2)}.$$

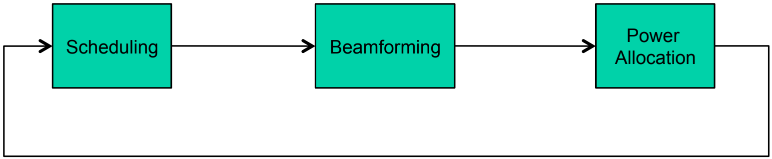
- Optimization problem:

$$\begin{aligned} \max \quad & \sum_{l,s,k} \log\left(\bar{R}_{lsk}\right) \\ \text{s.t.} \quad & R_{lsk} = \sum_{(b,n):k=f(l,s,b,n)} \log\left(1 + \mathrm{SINR}^n_{lsbk}\right) \end{aligned}$$

- This is a challenging (non-convex) problem

Divide and Conquer

- Three key steps, thus can iterate among them:
  - Power spectrum optimization
  - Coordinated beamforming
  - Proportionally fair scheduling
- Fixing two, the other step is a well-formulated problem.



- Heuristic
  - How to best do each step?
  - How well does it work?

Power Spectrum Optimization

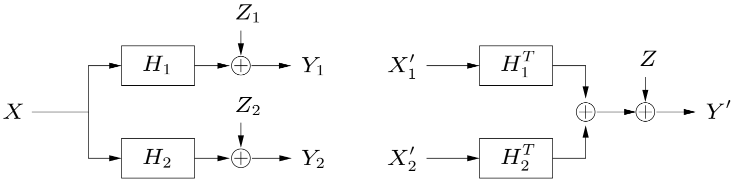
- Fixing scheduling and beamformers, the problem becomes:

$$\max \sum_{lsb} w_{lsb} \sum_{n} \log \left(1 + \frac{P_{lsb}^n |h_{lsb,lsk}^n|^2}{\Gamma(\sigma^2 + \sum_{(j,t,c)\neq(l,s,b)} P_{jtc}^n |h_{jtc,lsk}^n|^2)} \right)$$

- Many different approaches proposed in the literature:
  - Game-theory based approach (Ji-Huang '95 and many others)
  - Geometric programming (Chiang '02, '07)
  - SCALE algorithm (Papandriopoulos-Evans '06)
  - Pricing based approach (Huang-Berry-Honig '06, Yu '07)
  - Load-spillage (Hande-Rangan-Chiang '08)
  - Binary power control (Gjendemsjo-Gesbert-Oien-Kiani '08)
  - MAPEL/Polyblock optimization (Qian-Zhang-Huang '09, '10)
  - Iterative function evaluation (Dahrouj-Yu '10)
- No known efficient way to circumvent nonconvexity:
  - Fundamentally a difficult problem (Luo-Zhang '08).

Coordinated Beamforming

- Fundamental tool: Uplink-downlink duality



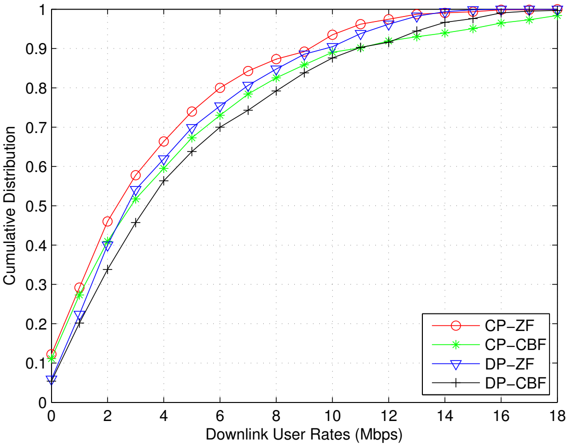
  - Single-cell: (Schubert-Boche '04, Bengtsson-Ottersten '02, Visotsky-Madhow '99, Wiesel et al '06, Song et al '07)
  - Multi-cell: (Rashid-Farrokhi et al '98, Dahrouj-Yu, '10)
- Use MMSE receive BF of the dual channel for transmit BF.
- Only works for power minimization for fixed SINR target.
  - Iterate between rate maximization and power minimization.

Scheduling

- Choose the best set of users to serve across multiple cells.
  - Full spatial multiplex: Schedule as many users as BS antennas
- Considerations:
  - Load balancing
  - Traffic shaping
  - Interference avoidance
- Downlink:
  - Interference is independent of scheduling
  - Venturino-Prasad-Wang '09, Stolyar-Viswanathan '09
- Uplink:
  - Discrete combinatoric optimization problem
  - Difficult problem, no known optimal solution
- Single-cell solution: (Yoo-Goldsmith '06)

How Well does It Work?

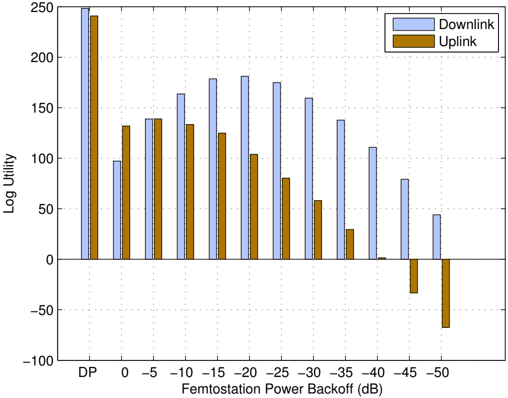
- 7-cell, 3-sector/cell, 4-antenna at BS, 2 at MS, full reuse



- BS-to-BS distance = 2.8km (Yu-Kwon-Shin '11)
  - 100% rate improvement for the 25th percentile user
  - 50% rate improvement for the 40th percentile user

Heterogeneous Topology

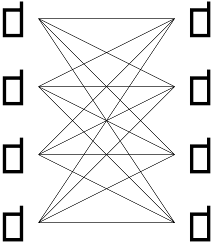
- 3-cell macro, 3-sector/cell, 3-femto/sector, 4 tx antenna



- Coordinated BF and power control outperform constant power backoff

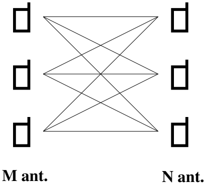
Can We do Better?

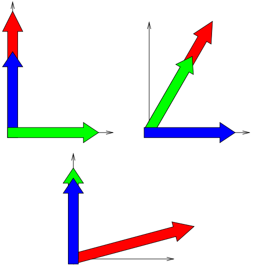
- What about interference alignment? (Cadambe-Jafar '08)



- For a K-user SISO interference channel coded across time or frequency dimensions.
- Degree-of-Freedom (DoF) per user is $\frac{K}{2}$
- For each user, the signal vector must not lie in the subspace spanned by interference.
- Alignment for cellular network: Suh-Ho-Tse '11,

Interference Alignment Through Linear Beamforming



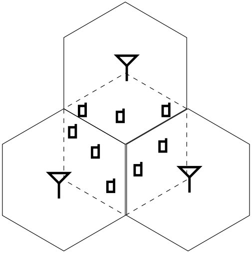


- What about without symbol extension?
- Consider K-user MIMO ($M \times N$) case
- Goal: deliver one data stream per user.
- $\mathbf{H}_{ij}$: channel between i^{th} tx. and j^{th} rx.
  $\mathbf{v}_j$: tx. beamformer at the j^{th} tx.
  $\mathbf{u}_j$: rx. beamformer at the i^{th} rx.
- We need:

$$\mathbf{u}_j^T \mathbf{H}_{ij} \mathbf{v}_i = 0 \text{ if } i \neq j$$
$$\mathbf{u}_j^T \mathbf{H}_{ij} \mathbf{v}_i \neq 0 \text{ if } i = j$$

- When is this possible?
  - Bezout's Theorem (Yetis, et al '10)
  - Counting # of eqs. vs # of unknowns

Interference Alignment for Cellular Networks



- Consider a 3-sector intersection:
  - M antennas/BS, N antennas/MS;
  - K users per sector.
- Assuming no symbol extensions, to align interference, we need:

$$\mathbf{u}_{pq}^T \mathbf{H}_{ipq} \mathbf{v}_{ij} = 0 \text{ if } i \neq p \text{ or } j \neq q$$
$$\mathbf{u}_{pq}^T \mathbf{H}_{ipq} \mathbf{v}_{ij} \neq 0 \text{ if } i = p \text{ and } j = q$$

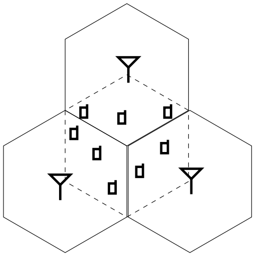
- Alignment is feasible only if: (Zhuang-Berry-Honig '12)

$$N + M \geq \sum_{i=1}^{3} K_i + 1$$

- For a 3-cell system with 4 ant. at the BS and 3 ant. at the user, only 2 users/cell can be scheduled (with no extension).
- More detailed analysis: Razaviyayn-Lyubeznik-Luo '11, Wang-Gou-Jafar '11, Bresler-Cartwright-Tse '11

What is the Right Number of Users to be Scheduled?

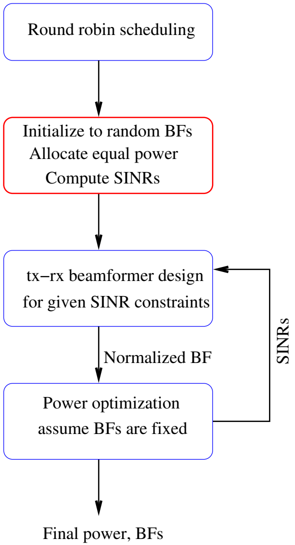
- Need system-level optimization to find out.
  - Fewer users open up more dimensions to ‘hide’ interference in;
  - More users can better utilize the available spatial dimensions.



- System setting:
  - Downlink, 3 cell sectors
  - 45 users/sector
  - One stream per scheduled user
  - 64 frequency tones
  - M tx antennas
  - N rx antennas

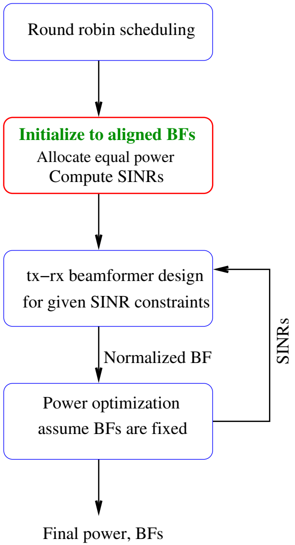
- Simplifying the setup:
  - Round-robin scheduling
  - Maximizing the sum rate
  - Iterate between duality-based BF and power optimizations

Flowchart for Optimization: Full Spatial Multiplexing



- Schedule full set of users.
- Initialize with random BF, equal power.
- Use duality-based algorithm to find the BF iteratively
- Use interior-point method for account for per-BS sum power constraint

Flowchart for Optimization: What about Alignment?



- Only schedule as many users as alignment allows.
- Many sets of aligned BFs exist.
- Aligned BFs neglect direct channel.
- We further use duality-based tx-rx BF design to refine the BF design.

Computing Aligned Beamformers

- Use algorithm of Gomadam-Cadambe-Jafar '08 (see also Peters-Heath '09)
- Interference at the j^{th} user in the i^{th} cell is given by:

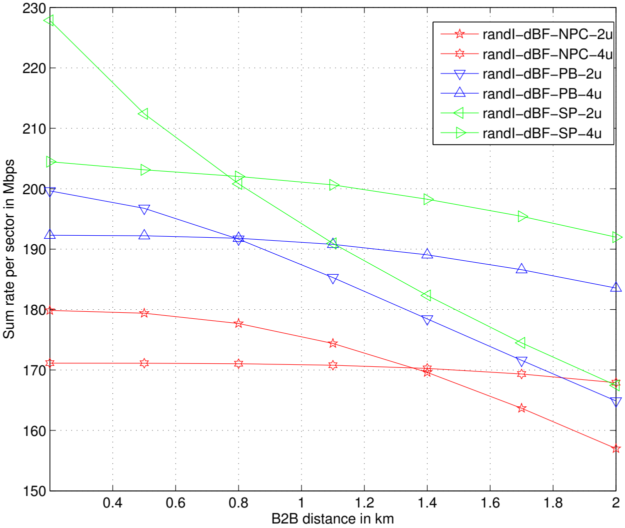
$$I_{ij} = \sum_{(p,q)\neq(i,j)} \mathbf{u}_{ij}^H \mathbf{H}_{pij} \mathbf{v}_{pq}$$

- Covariance of I_{ij} is given by:

$$Cov(I_{ij}) = \mathbf{u}_{ij}^H \left(\sum_{(p,q)\neq(i,j)} \mathbf{H}_{pij} \mathbf{v}_{pq} \mathbf{v}_{pq}^H \mathbf{H}_{pij}^H \right) \mathbf{u}_{ij} \triangleq \mathbf{u}_{ij}^H \mathbf{Q}_{ij} \mathbf{u}_{ij}$$

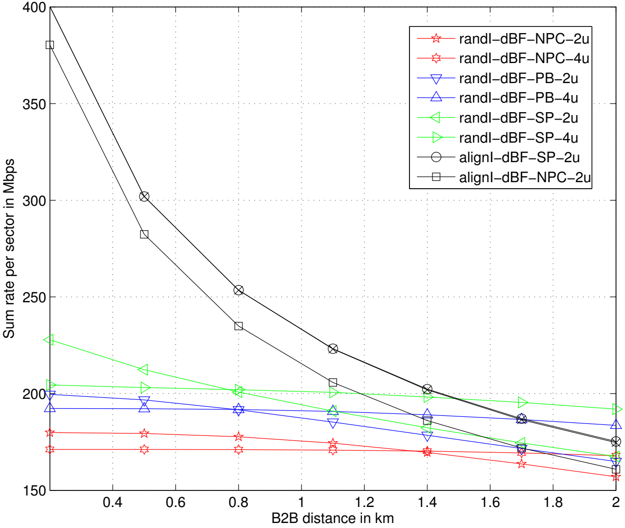
- To minimize interference I_{ij}, set $\mathbf{u}_{ij}$ to $\nu_L(\mathbf{Q}_{ij})$, where $\nu_L(\mathbf{Q}_{ij})$ is the eigenvector of $\mathbf{Q}_{ij}$ with the smallest eigenvalue.
- Update all $\mathbf{u}_{ij}$; use reciprocity to update $\mathbf{v}_{ij}$ similarly.
- Repeat until convergence.

What is the Optimal Number of Users to Schedule?



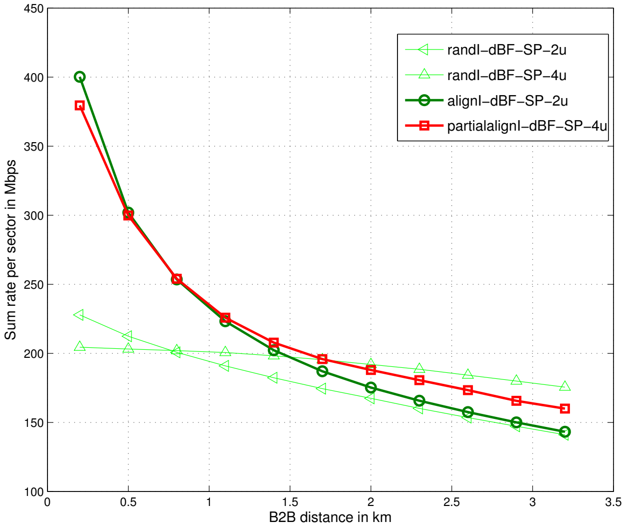
- M=4, N=3
- 2 or 4 users/cell/tone
- power opt:
  - no power control
  - per beam const.
  - sum power const.
- Initialization:
  - Random BF
  - Uniform power init.
- Observations:
  - fewer users better at smaller dist.
  - Crossover point shifts left

At Convergence, are BFs Aligned? Are Aligned BFs Easy to Find?



- M=4, N=3
- 2 or 4 users/cell/tone
- power opt:
  no power control
  per beam const.
  sum power const.
- Initialization:
  Random/aligned BF
  Uniform power init.
- Observations:
  – aligned initialization significantly better
  – aligned BF hard to find w/ random init.
  – # users scheduled plays important role
- Similar conclusion by Zhuang-Berry-Honig'12

Optimization of the '# of Users to Schedule'



- M=4, N=3
- 2 or 4 users/cell/tone
- Initialization:
  Random/aligned BFs
  Partially aligned BFs
  Uniform power init.
- power opt:
  sum power const.
- BF design:
  duality tx-rx BF
- Observations:
  – partial align init. circumvents sch. issue
  – some performance loss at higher distances

Remarks

- There are substantial benefits for system-level optimization
  - for both cellular and (especially) heterogeneous networks.
- Optimization is also quite difficult:
  - Nonconvexity of power optimization is difficult to circumvent;
  - Beamforming is intricately connected to power control;
  - Discrete nature of user scheduling is hard.
- Interference alignment opens up a new dimension
  - What is the optimal number of users to schedule?
  - Aligned solutions are not unique, how to identify the best one? (Schmidt-Utschick-Honig '10, Santamaria-Gonzalez-Heath-Peters '10)
- Many practical issues:
  - Channel estimation and feedback.
  - Rated-limited cooperation in network MIMO.

Talks

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