# Coordinated Beamforming for MISO Interference Channels: Achievable Rate and Distributed Algorithm

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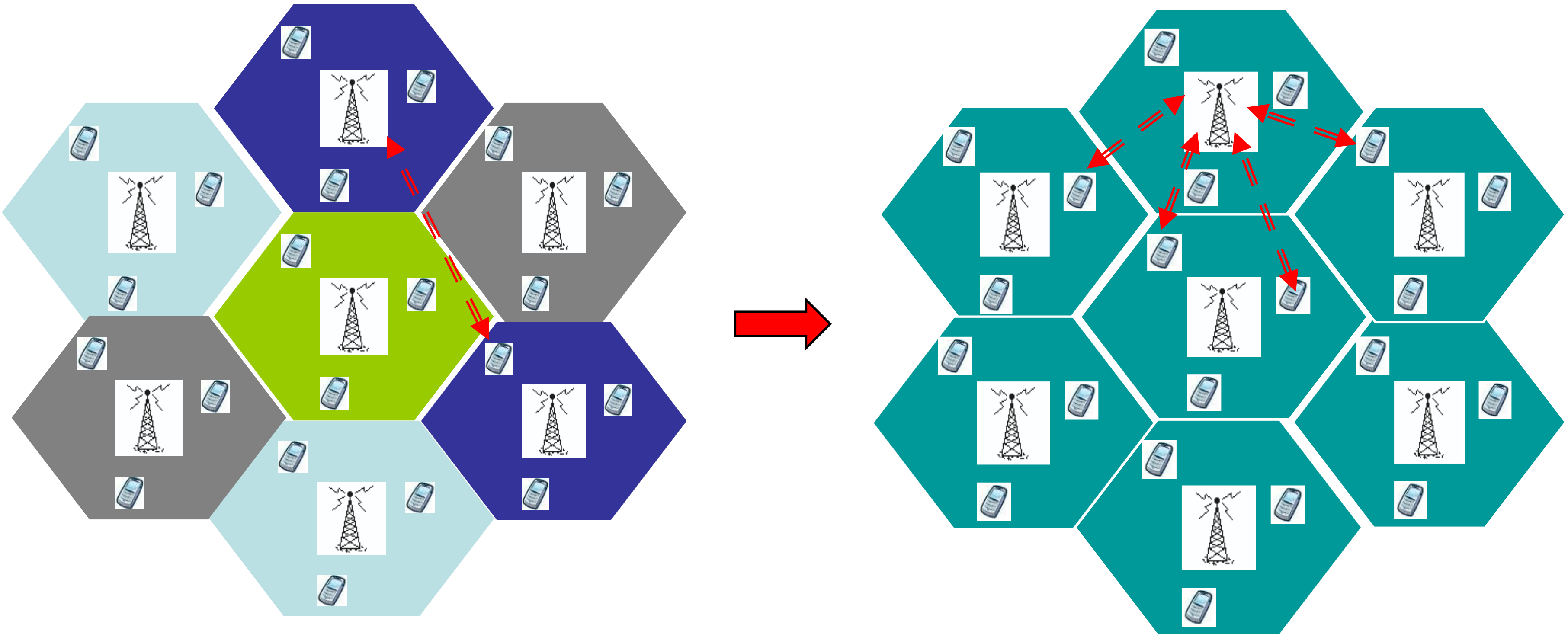
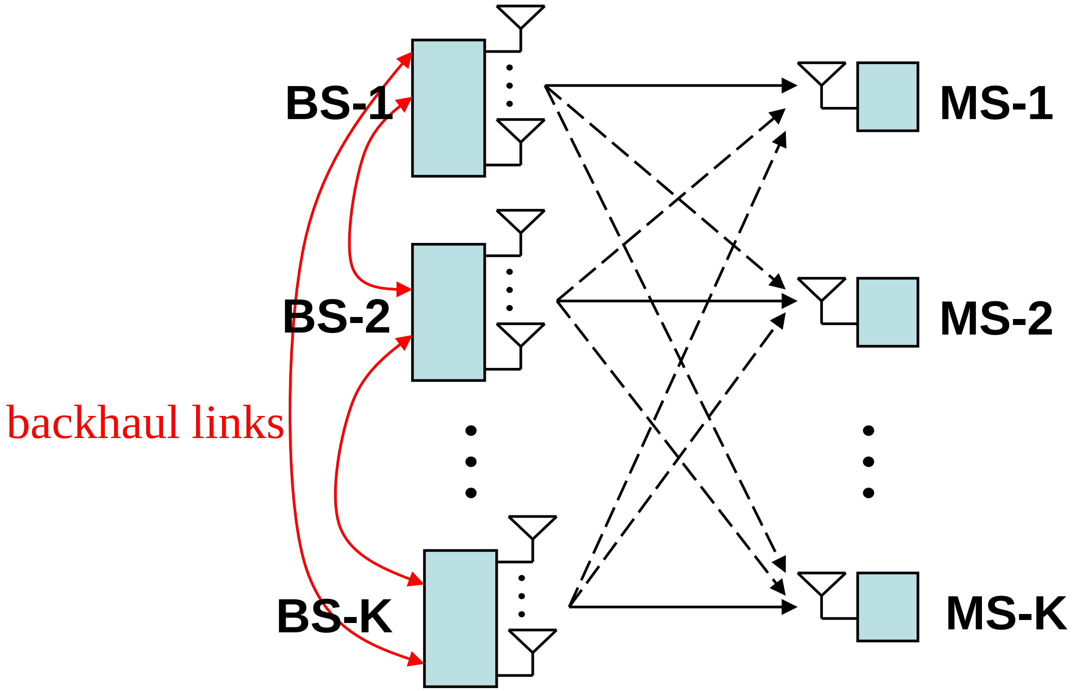
# A New Look at Cellular Networks



- **Future trends: universal/opportunistic frequency reuse**
  - ❑ Pros: more abundant bandwidth allocation
  - ❑ Cons: more severe inter-cell interference (ICI)
  - ❑ Need more sophisticated cooperative signal processing among BSs

# Multi-Cell Cooperative MIMO (Downlink)




- **Network MIMO/CoMP (centralized approach)**
  - ❑ Global transmit message sharing across all BSs
  - ❑ ICI utilized for coherent transmission
  - ❑ MIMO Broadcast Channel (MIMO-BC) with per-BS power constraints
- **Interference Coordination (decentralized approach)**
  - ❑ Local transmit message known at each BS
  - ❑ ICI controlled to best effort
  - ❑ MIMO Interference Channel (MIMO-IC) *or* partially interfering MIMO-BC
- Other approaches
  - ❑ *hybrid model, MIMO X channel, finite backhaul capacity,…*

# Network MIMO: Capacity Upper Bound

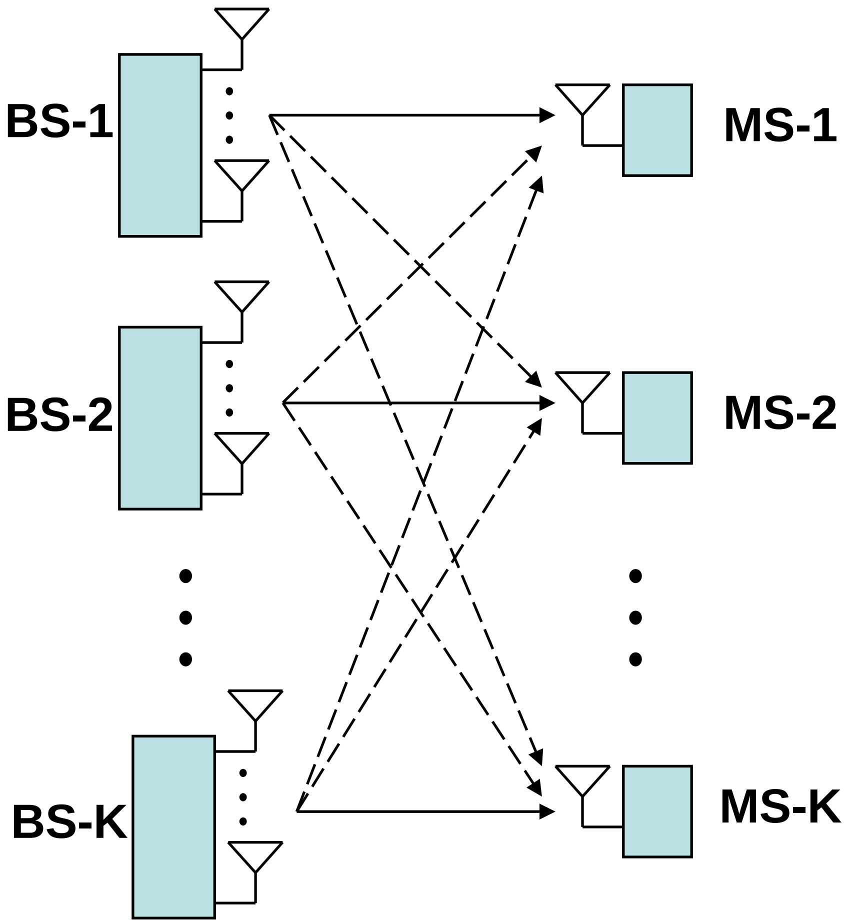
## MIMO-BC with per-BS power constraints

- Nonlinear dirty-paper precoding (DPC)
  - Optimality of DPC [CaireShamai03] [ViswanathTse03] [YuCioffi04] [WeingartenSteinbergShamai06]
  - DPC region characterization (e.g., via weighted sum-rate maximization)
    - BC-MAC duality for sum-power constraint [VishwanathJindalGoldsimith03]
    - Min-Max duality for sum-/per-antenna power constraints [YuLan07]
    - Generalized BC-MAC duality for arbitrary linear transmit power constraints **[ZhangZhangLiangXinPoor12]**
- Linear zero-forcing (ZF) *or* block diagonalization (BD) precoding
  - Sum-power constraint (MIMO-BC): [WongMurchLetaief03], [SpencerSwindlehurstHaardt04]
  - Per-antenna power constraint (MISO-BC): [WieselEldarShamai08] [HuhPapadopoulosCaire09]
  - General linear transmit power constraints (MIMO/MISO-BC): **[Zhang10]**

**L. Zhang, R. Zhang, Y. C. Liang, Y. Xin, and H. V. Poor, "On the Gaussian MIMO BC-MAC duality with multiple transmit covariance constraints," *IEEE Transactions on Information Theory,* April 2012.**

**R. Zhang, "Cooperative multi-cell block diagonalization with per-base-station power constraints," *IEEE Journal on Selected Areas in Communications*, Dec. 2010**.

# Interference Coordination: MISO-IC




## Assumptions:

- one active user per cell
- interference treated as noise
- symmetric (proper) complex Gaussian input

## Signal Model:

$$y_k = \boldsymbol{h}_{kk}^H \boldsymbol{x}_k + \sum_{j \neq k}^{K} \boldsymbol{h}_{jk}^H \boldsymbol{x}_j + z_k, \quad k = 1, \ldots, K$$

- $y_k$: received signal at the $k$th MS
- $\boldsymbol{x}_k \in \mathbb{C}^{M_k \times 1}$: transmitted signal from the $k$th BS, $M_k \geq 1$
- $\boldsymbol{h}_{kk}^H \in \mathbb{C}^{1 \times M_k}$: direct-link channel for the $k$th BS-MS pair
- $\boldsymbol{h}_{jk}^H \in \mathbb{C}^{1 \times M_j}$: cross-link channel from the $j$th BS to $k$th MS, $j \neq k$
- $z_k$: receiver noise at the the $k$th MS, $z_k \sim \mathcal{CN}(0, \sigma_k^2)$
- $\boldsymbol{x}_k$'s are independent over $k$: no message sharing among BSs
- $\boldsymbol{S}_k \triangleq \mathbb{E}[\boldsymbol{x}_k \boldsymbol{x}_k^H]$: transmit covariance matrix for the $k$th BS, $\boldsymbol{S}_k \succeq 0$

# Related Work

- Gaussian Interference Channel
  - Capacity region unknown in general
  - Best known achievability scheme: [HanKobayashi81]
  - Capacity within 1-bit: [EtkinTseWang08]

- Pragmatic Approach (interference treated as noise)
  - Interference alignment [Jafar *et al.*]
    - DoF optimality at high SNR
    - New ingredients: *improper complex Gaussian signaling, time symbol extension, non-separability of parallel Gaussian ICs*
  - MISO-IC (finite-SNR, proper Gaussian input)
    - Achievable rate region characterization [JorswieckLarssonDanev08]
    - Power minimization with SINR constraints [DahroujYu10]
    - Optimality of beamforming (rank-one transmit covariance matrix) [ShangChenPoor11]
  - Weighted sum-rate maximization (WSRMax) via "Monotonic Optimization"
    - SISO-IC [QianZhangHuang09]
    - MISO-IC [JorswieckLarsson10] [BjornsonZhengBengtssonOttersten12]
  - WSRMax for MIMO-IC
    - [PetersHeath10] [RazaviyaynSanjabiLuo12]….

# Pareto Optimal Rates in MISO-IC

- Achievable user rate (with interference treated as noise)

$$R_k(\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K) = \log\left(1 + \frac{\boldsymbol{h}_{kk}^H \boldsymbol{S}_k \boldsymbol{h}_{kk}}{\sum_{j \neq k} \boldsymbol{h}_{jk}^H \boldsymbol{S}_j \boldsymbol{h}_{jk} + \sigma_k^2}\right), \quad k = 1, \ldots, K$$

- Achievable rate region (prior to time sharing)

$$\mathcal{R} \triangleq \bigcup_{\{\boldsymbol{S}_k\}: \mathrm{Tr}(\boldsymbol{S}_k) \leq P_k, \forall k} \left\{(r_1, \ldots, r_K) : 0 \leq r_k \leq R_k(\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K), k = 1, \ldots, K\right\}$$

- Pareto rate optimality

**Definition**: A rate-tuple $(r_1, \ldots, r_K)$ is *Pareto optimal* if there is no other rate-tuple $(r_1', \ldots, r_K')$ with $(r_1', \ldots, r_K') \geq (r_1, \ldots, r_K)$ and $(r_1', \ldots, r_K') \neq (r_1, \ldots, r_K)$ (the inequality is component-wise).

# Weighted Sum Rate Maximization

$$
\begin{aligned}
(\mathrm{WSRMax}):\ \underset{\boldsymbol{S}_1,\cdots,\boldsymbol{S}_K}{\texttt{max.}}\quad & \sum_{k=1}^{K} w_k \log\left(1+\frac{\boldsymbol{h}_{kk}^H\boldsymbol{S}_k\boldsymbol{h}_{kk}}{\sum_{j\neq k}\boldsymbol{h}_{jk}^H\boldsymbol{S}_j\boldsymbol{h}_{jk}+\sigma_k^2}\right)\\
\texttt{s.t.}\quad & \mathrm{Tr}(\boldsymbol{S}_k)\le P_k,\ \forall k\\
& \boldsymbol{S}_k \succeq \mathbf{0},\ \forall k
\end{aligned}
$$

Non-convex problem, cannot be solved directly by convex optimization techniques

# SINR Feasibility Problem

Assuming transmit beamforming *i.e.* $\boldsymbol{S}_k = \boldsymbol{v}_k \boldsymbol{v}_k^H, \forall k$

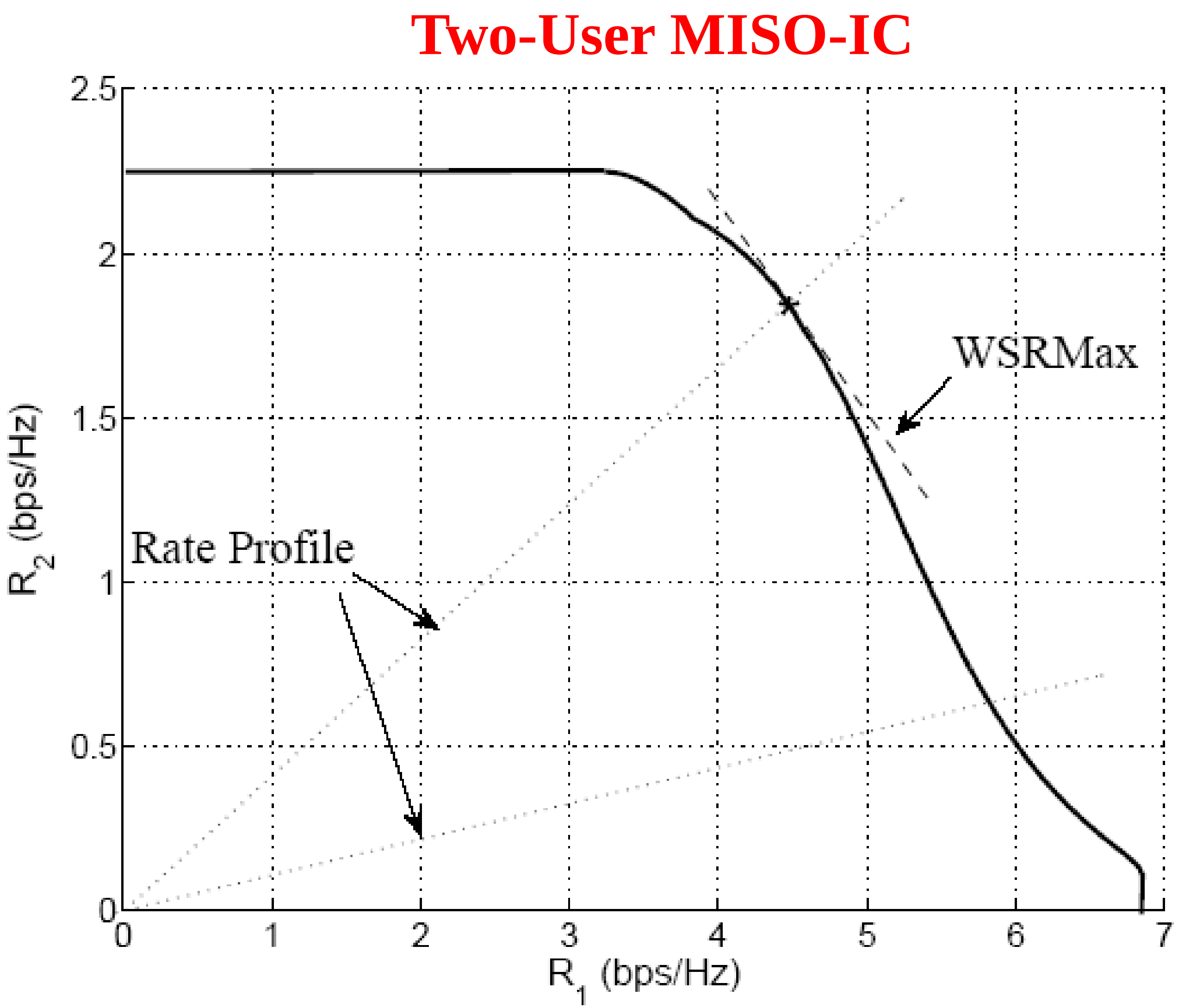
$$
\begin{aligned}
(\mathrm{SINR-Feas.}): \quad & \mathtt{find} \quad \{\boldsymbol{v}_k\} \\
\mathtt{s.t.} \quad & \frac{1}{\bar{\gamma}_k} \|\boldsymbol{h}_{kk}^H \boldsymbol{v}_k\|^2 \geq \sum_{j \neq k} \|\boldsymbol{h}_{jk}^H \boldsymbol{v}_j\|^2 + \sigma_k^2, \quad \forall k \\
& \|\boldsymbol{v}_k\|^2 \leq P_k, \forall k
\end{aligned}
$$

Convex problem, can be solved efficiently via a convex Second Order Cone Programming (SOCP) feasibility problem

**Question: Can we solve WSRMax via SINR-Feas. problem for ICs?**

**L. Liu, R. Zhang, and K. C. Chua, "Achieving global optimality for weighted sum-rate maximization in the K-user Gaussian interference channel with multiple antennas," *IEEE Transactions on Wireless Communications*, May 2012. (also see** [BjornsonZhengBengtssonOttersten12])

# Rate-Profile Approach



# Sum-Rate Maximization with Rate-Profile Constraint

Given a rate-profile vector $\boldsymbol{\alpha} = [\alpha_1, \ldots, \alpha_K] \succeq 0$, $\sum_{k=1}^{K} \alpha_k = 1$

$$\begin{aligned} \max_{R_{\text{sum}}, \{\boldsymbol{w}_k\}} \quad & R_{\text{sum}} \\ \text{s.t.} \quad & \log\left(1 + \gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)\right) \geq \alpha_k R_{\text{sum}}, \quad \forall k \\ & \|\boldsymbol{w}_k\|^2 \leq P_k, \quad \forall k \end{aligned}$$

$$\begin{aligned} \text{find} \quad & \{\boldsymbol{w}_k\} \\ \text{s.t.} \quad & \log\left(1 + \gamma_k(\boldsymbol{w}_1, \ldots, \boldsymbol{w}_K)\right) \geq \alpha_k r_{\text{sum}}, \quad \forall k \\ & \|\boldsymbol{w}_k\|^2 \leq P_k, \quad \forall k \end{aligned}$$

**SINR-Feas. Problem**

Non-convex problem, but efficiently solvable via a sequence of convex SINR-Feas. Problems

**R. Zhang and S. Cui, "Cooperative interference management with MISO beamforming," *IEEE Transactions on Signal Processing,* Oct. 2010.**

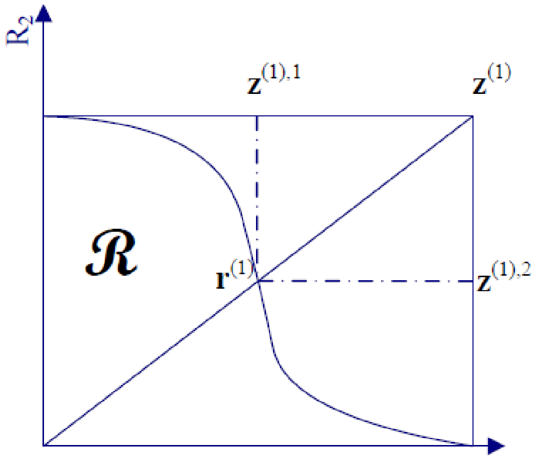
# Monotonic Optimization

Key observation: maximize WSR in MISO-IC rate region directly!

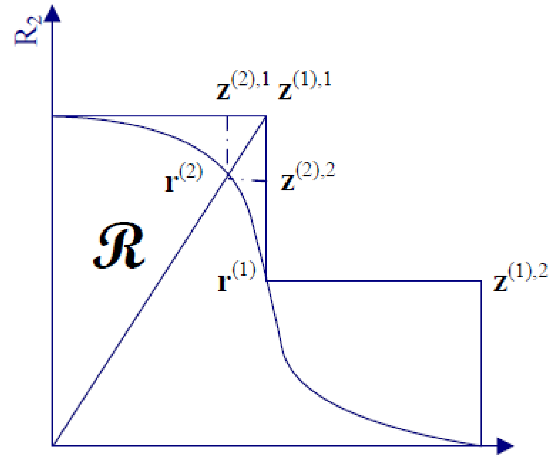
$$(\text{WSRMax}): \quad \underset{\boldsymbol{r}=[R_1,\ldots,R_K]}{\texttt{max.}} \; U(\boldsymbol{r}) := \sum_{k=1}^{K} \mu_k R_k$$
$$\texttt{s.t.} \quad \boldsymbol{r} \in \mathcal{R}$$

monotonic optimization problem (maximize a strictly increasing function over a "normal" set), thus solvable by *e.g.* the "outer polyblock approximation" algorithm

# Outer Polyblock Approximation

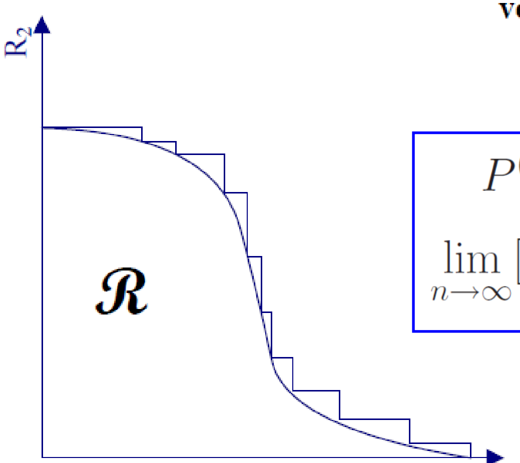



The polyblock $P^{(1)}$ with vertex $z^{(1)}$




The polyblock $P^{(2)}$ with vertices $z^{(1),1}$, $z^{(1),2}$

. . .




The polyblock $P^{(n)}$ which approaches the rate region with increasing n

$$P^{(1)} \supset P^{(2)} \supset \cdots \supset \mathcal{R},$$

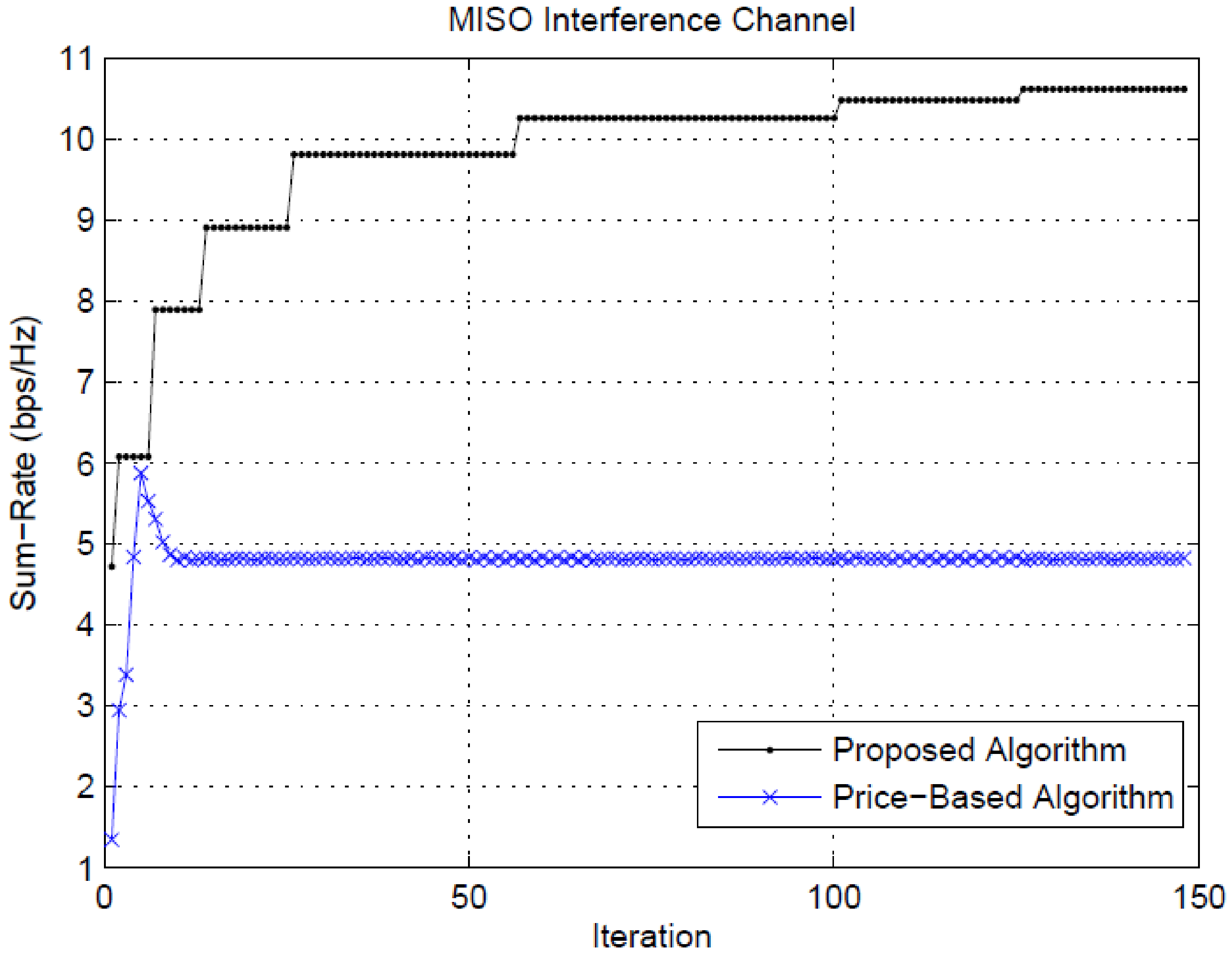
$$\lim_{n\to\infty} \left[ \max_{\boldsymbol{r}\in P^{(n)}} U(\boldsymbol{r}) \right] = \max_{\boldsymbol{r}\in\mathcal{R}} U(\boldsymbol{r})$$

- Guaranteed convergence
- Controllable accuracy
- Complexity: ???
- **Key step** in each iteration: Find intersection point with Pareto boundary given a rate profile, which is solved by Sum-Rate Maximization with Rate-Profile Constraint

Rate Profile + Monotonic Optimization solves WSRMax for MISO-IC

# Numerical Example

- Baseline scheme: "price-based" algorithm [Schmidt *et al*.09]
- MISO-IC: $M_k=2$, $K=4$, *i.i.d. Rayleigh fading*, $SNR_k=3$, $w_k=1$

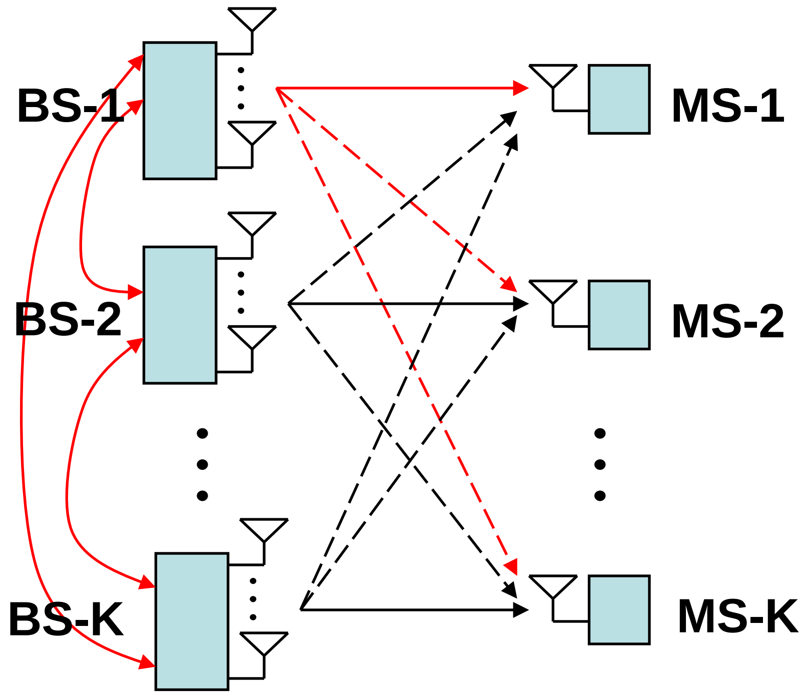


# Recap

- Pareto rate characterization for Gaussian MISO-IC
  - non-convex problem in general
  - rate maximization with user fairness: rate profile *vs.* WSRMax
  - rate-profile: polynomial complexity, scalable with # of users
  - WSRMax: unknown complexity , non-scalable with # of users
  - similar results extendible to SISO-IC *or* SIMO-IC, *but* not MIMO-IC

- Developed a new framework for *non-convex* utility optimization in multiuser networks (with conflict interests) based on rate profile and monotonic optimization, provided
  - problem size not so large
  - finding intersection points with Pareto boundary is efficiently solvable

# Distributed Beamforming for MISO-IC

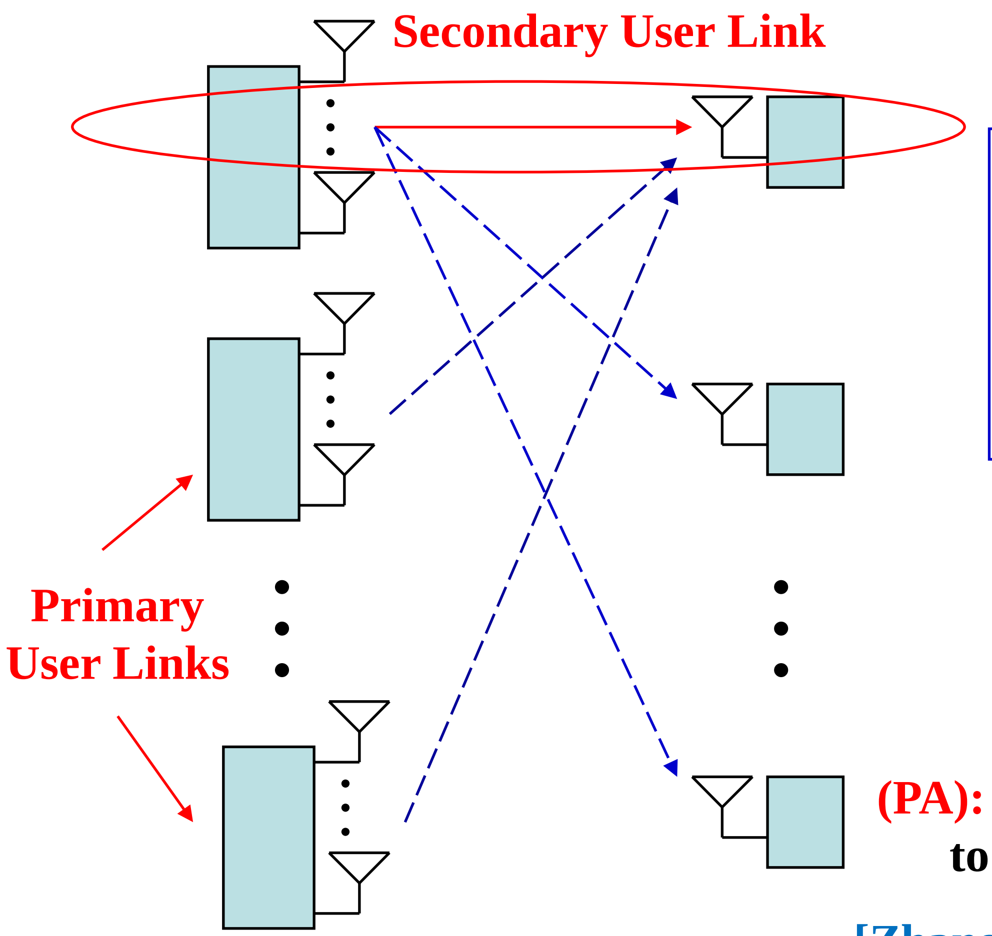



- Distributed Beamforming
  - low-rate information exchange across BSs
  - only “local” channel knowledge available at each BS

**Question: Can we archive distributed (Pareto rate) optimal beamforming?**

**R. Zhang and S. Cui, “Cooperative interference management with MISO beamforming,” *IEEE Transactions on Signal Processing,* Oct. 2010.**

# Exploiting a Relationship between MISO-IC and MISO Cognitive Radio (CR) Channel




- **For the *k*th MISO CR link:**

$$C_k(\Gamma_k) := \max_{S_k} . \ \log\left(1 + \frac{h_{kk}^H S_k h_{kk}}{\sum_{j \neq k} \Gamma_{jk} + \sigma_k^2}\right)$$
$$\text{s.t.} \ h_{kj}^H S_k h_{kj} \leq \Gamma_{kj}, \ \forall j \neq k$$
$$\text{Tr}(S_k) \leq P_k, \ S_k \succeq 0$$

**Interference power to other MSs**

**Interference power from other BSs**

**(PA):** **"Cognitive beamforming (CB)" problem subject to "Interference Temperature (IT)" constraints**

**[ZhangLiang08]**

R. Zhang and Y. C. Liang, "Exploiting multi-antennas for opportunistic spectrum sharing in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, 2008.

# Optimal Cognitive Beamforming (CB)

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**Theorem**: The optimal solution for $\boldsymbol{S}_k$ in (PA) is *rank-one*, i.e., $\boldsymbol{S}_k = \boldsymbol{w}_k \boldsymbol{w}_k^H$, and

$$\boldsymbol{w}_k = \left( \sum_{j \neq k} \lambda_{kj} \boldsymbol{h}_{kj} \boldsymbol{h}_{kj}^H + \lambda_{kk} \boldsymbol{I} \right)^{-1} \boldsymbol{h}_{kk} \sqrt{p_k}$$

where $\lambda_{kj}$, $j \neq k$, and $\lambda_{kk}$ are non-negative constants (solutions for the dual problem of (PA)); and $p_k$ is given by

$$p_k = \left( \frac{1}{\ln 2} - \frac{\sum_{j \neq k} \Gamma_{jk} + \sigma_k^2}{\|\boldsymbol{A}_k \boldsymbol{h}_{kk}\|^2} \right)^+ \frac{1}{\|\boldsymbol{A}_k \boldsymbol{h}_{kk}\|^2}$$

where $\boldsymbol{A}_k \triangleq \left( \sum_{j \neq k} \lambda_{kj} \boldsymbol{h}_{kj} \boldsymbol{h}_{kj}^H + \lambda_{kk} \boldsymbol{I} \right)^{-1/2}$ and $(x)^+ \triangleq \max(0, x)$.

A semi-closed-form solution efficiently solvable by an iterative inner-outer-loop algorithm

# Interference Temperature (IT) Approach to Characterize MISO-IC Pareto Boundary

**Proposition**: For any rate-tuple $(R_1, \ldots, R_K)$ on the Pareto boundary of the MISO-IC rate region, which is achievable with a set of transmit covariance matrices, $\boldsymbol{S}_1, \ldots, \boldsymbol{S}_K$, there is a corresponding interference-power/interferecne-temperature constraint vector, $\boldsymbol{\Gamma} \geq 0$, with $\Gamma_{kj} = \boldsymbol{h}_{kj}^H \boldsymbol{S}_k \boldsymbol{h}_{kj}, \forall j \neq k, j \in \{1, \ldots, K\}$, and $k \in \{1, \ldots, K\}$, such that $R_k = C_k(\boldsymbol{\Gamma}_k), \forall k$, and $\boldsymbol{S}_k$ is the optimal solution of (PA) for the given $k$.

- A new parametrical characterization of MISO-IC Pareto boundary in terms of BSs' mutual IT levels, which constitute a lower-dimensional manifold than transmit covariance matrices

- Optimality of beamforming for MISO-IC is proved
  (see an alternative proof in [ShangChenPoor11])

# Necessary Condition of Pareto Optimality

**Theorem**: For an arbitrarily chosen $\boldsymbol{\Gamma} = [\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_K] \geq 0$, if the optimal rate values for all $k$'s, $C_k(\boldsymbol{\Gamma}_k)$'s, are Pareto-optimal on the boundary of the MISO-IC rate region, then for any pair of $(i,j), i \in \{1, \ldots, K\}, j \in \{1, \ldots, K\}$, and $i \neq j$, it must hold that $|\boldsymbol{D}_{ij}| = 0$, where

$$\boldsymbol{D}_{ij} = \begin{bmatrix} \frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ij}} & \frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ji}} \\ \frac{\partial C_j(\boldsymbol{\Gamma}_j)}{\partial \Gamma_{ij}} & \frac{\partial C_j(\boldsymbol{\Gamma}_j)}{\partial \Gamma_{ji}} \end{bmatrix} := \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

where

$$\frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ij}} = \lambda_{ij}$$

$$\frac{\partial C_i(\boldsymbol{\Gamma}_i)}{\partial \Gamma_{ji}} = \frac{-\boldsymbol{h}_{ii}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ii}}{\ln 2(\sum_{l \neq i} \Gamma_{li} + \sigma_i^2)(\sum_{l \neq i} \Gamma_{li} + \sigma_i^2 + \boldsymbol{h}_{ii}^H \boldsymbol{S}_i^\star \boldsymbol{h}_{ii})}$$

# Distributed Beamforming based on CB and "Active IT Control"

- BS **pair-wise** IT update:

$$[\Gamma_{ij}, \Gamma_{ji}]^T \leftarrow [\Gamma_{ij}, \Gamma_{ji}]^T + \delta_{ij} \cdot \boldsymbol{d}_{ij}$$

step size

fairness control

where $\boldsymbol{d}_{ij} = \text{sign}(ad - bc) \cdot [\alpha_{ij} d - b, a - \alpha_{ij} c]^T$

- **Distributed** coordinated beamforming for MISO-IC:

Initialize $\boldsymbol{\Gamma} \geq 0$ in the network

BS $k$ sets $\boldsymbol{w}_k$ by solving (PA) with the given $\boldsymbol{\Gamma}_k$, $k = 1, \ldots, K$

Repeat

  For $i = 1, \ldots, K$, $j = 1, \ldots, K$, $j \neq i$,

    BS $i$ computes $a$ and $b$ in $\boldsymbol{D}_{ij}$ with the given $\boldsymbol{\Gamma}_i$

    BS $j$ computes $d$ and $c$ in $\boldsymbol{D}_{ij}$ with the given $\boldsymbol{\Gamma}_j$

    BS $i$ sends $a$ and $b$ to BS $j$

    BS $j$ sends $c$ and $d$ to BS $i$

    BS $i$ ($j$) computes $\boldsymbol{d}_{ij}$, and updates $\Gamma_{ij}$ and $\Gamma_{ji}$

    BS $i$ ($j$) resets $\boldsymbol{w}_i$ ($\boldsymbol{w}_j$) by solving (PA) with the updated $\boldsymbol{\Gamma}_i$ ($\boldsymbol{\Gamma}_j$)

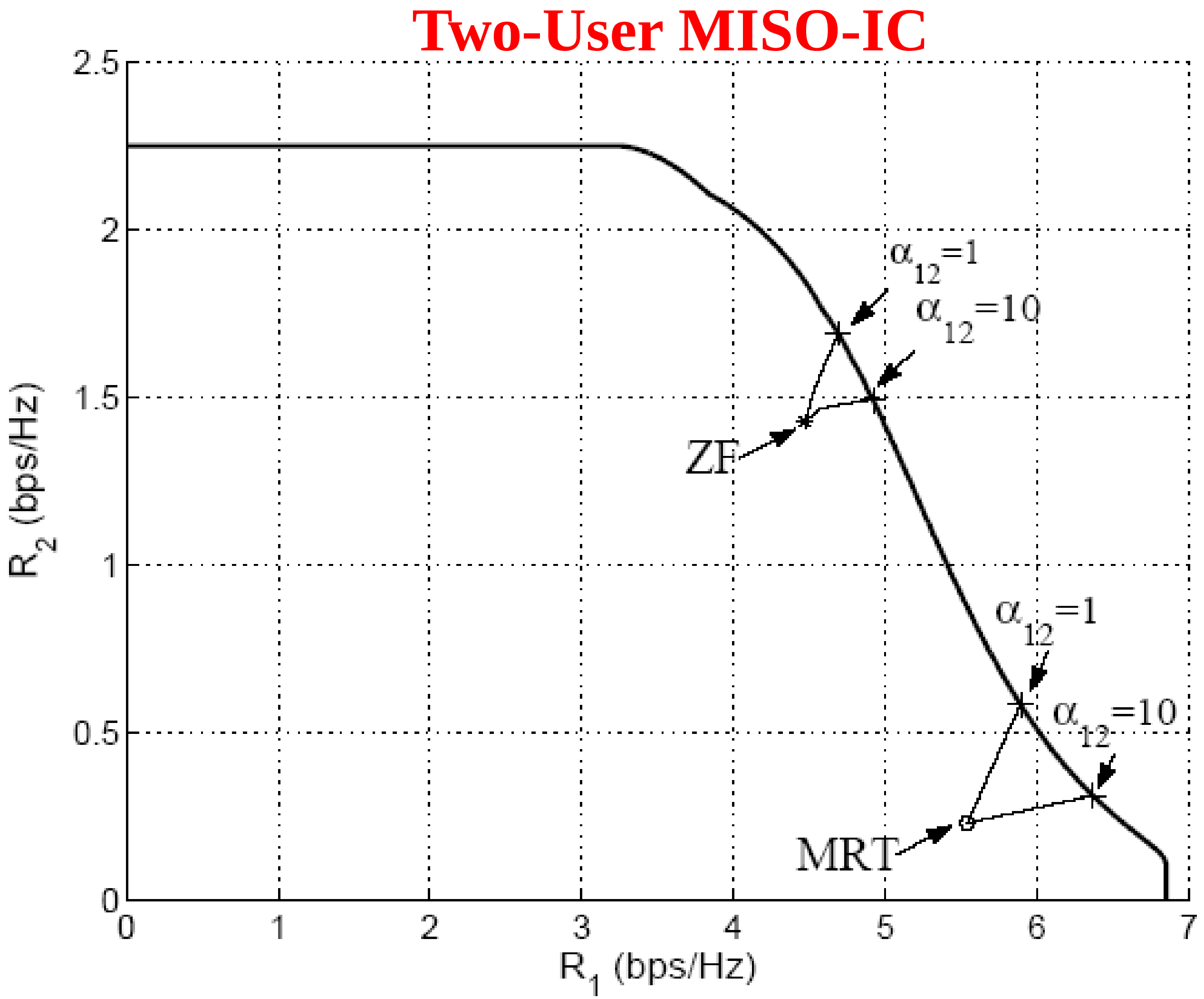
  End For

Until $|\boldsymbol{D}_{ij}| = 0, \forall i \neq j$.

Exchange two scalars per update

# Numerical Example

- MISO-IC: $M_1 = M_2 = 3$, $K=2$, *i.i.d. Rayleigh fading*, $SNR_1 = 5$, $SNR_2 = 1$

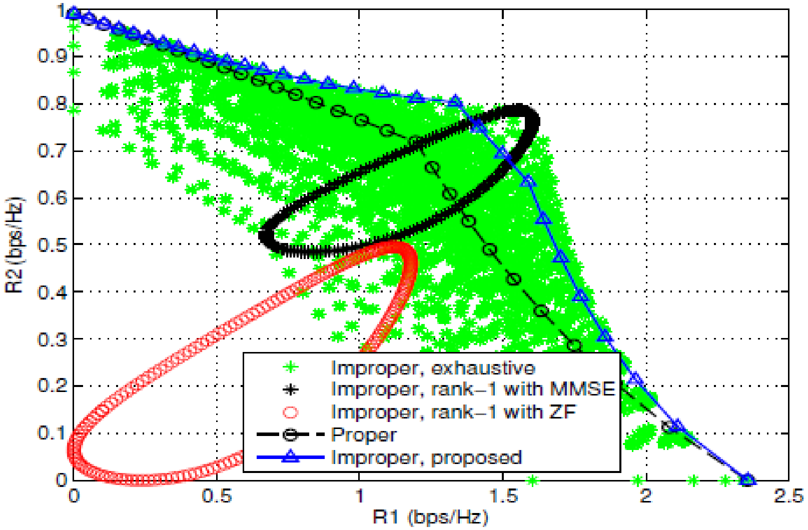


# Concluding Remark

- **Transmit optimization for MISO-IC**
  - WSRMax characterization (rate profile + monotonic optimization)
  - optimal distributed beamforming (cognitive beamforming + active IT control)

- **Next Step: improper Gaussian signaling (submitted to Asilomar 2012)**

$$R_r = \underbrace{\log\left(1 + \frac{|h_{rr}|^2 C_{x_r}}{\sigma^2 + |h_{r\bar{r}}|^2 C_{x_{\bar{r}}}}\right)}_{R_r^{\text{proper}}(C_{x_1}, C_{x_2})} + \frac{1}{2}\log\frac{1 - C_{y_r}^{-2}|\widetilde{C}_{y_r}|^2}{1 - C_{s_r}^{-2}|\widetilde{C}_{s_r}|^2}.$$




two-user SISO-IC