Spatial Interference Management for Dense Wireless Networks: Alignment and Other Techniques

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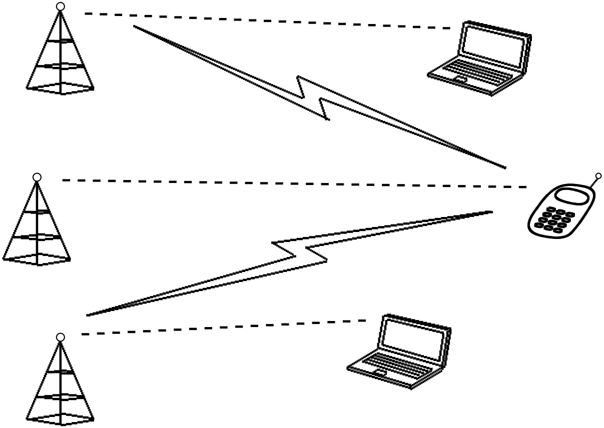
Current Systems are Interference-Limited

traffic growth + expensive spectrum = frequency reuse + interference

(the equation faced by wireless communication engineers...)



Goals of this Presentation

Focus on the Gaussian MIMO Interference Channel (MIMO-IC)

- Analyze interference alignment (IA) performance for
    - non-asymptotic SNR
    - many users, many antennas
- Compare to distributed Tx covariance optimization

NOT Covered Here

- Numerous derivative works sparked by IA:
    - asymmetric complex signaling
    - lattice alignment
    - rational dimension alignment...
- Robustness w.r.t. imperfect CSI
    - some clues available from [Tresch,Guillaud ICC'09]
- Stochastic geometry-based analysis of IA in dense networks
    - Per-Cluster IA addressed in [Tresch,Guillaud ISIT'10]

Recent Focus on the K-User MIMO-IC

K-User MIMO Gaussian Interference Channel

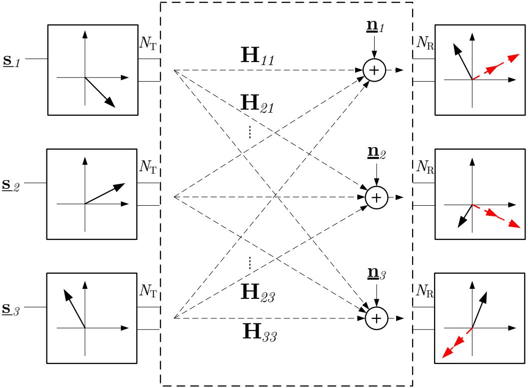
$$\underline{y}_i = \sum_{j=1}^{K} \mathbf{H}_{ij}\underline{x}_j + \underline{n}_i \quad \forall i = 1 \dots K.$$

- Rx *i* is interested in the message of Tx *i* only

Interference Alignment

- [Gomadam,Cadambe,Jafar IT'08 , Cadambe,Jafar IT'08] proposes
  - low rank (less than channel rank) linear precoding
  - create a subspace without interference *at the receiver*

Interference Alignment in Pictures



- $\underline{x}_i = \mathbf{V}_i \underline{s}_i \quad \forall i$, where $\mathbf{V}_i$ is tall, and $\underline{s}_i$ contains the symbols to transmit
- interference $\sum_{j \neq i} \mathbf{H}_{ij} \mathbf{V}_j \underline{s}_j$ does not occupy all receive dimensions

Why is IA Attractive ?

- Simple formulation: find matrices $\mathbf{V}_i$ and $\mathbf{U}_i$ (resp. $N_{Ti} \times d_i$ and $N_{Ri} \times d_i$)

$$\text{s.t.} \quad \begin{cases} \mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ij} \mathbf{V}_j & = & 0, \ \forall j \neq i \\ \operatorname{rank}\left(\mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{V}_i\right) & = & d_i. \end{cases}$$

- No complex multiuser codes, interference treated as noise
- Achievable rate per user does not saturate at high SNR

$$\lim_{SNR \to +\infty} \frac{C_i(SNR)}{\log SNR} = d_i$$

  (a.s. under reasonable channel assumptions)
  - d_i interference-free signaling dimensions, or degree of freedom (DoF) available to user i.
- "each user gets half of the cake"
  - In symmetric systems with square channels ($N_{\mathrm{R}} = N_{\mathrm{T}} = N$, $d_i = d \,\forall i$), IA is feasible for d up to $\frac{N}{2}$.
  - Point-to-Point MIMO: $\lim_{SNR \to +\infty} \frac{C(SNR)}{\log SNR} = N$ a.s.

Limitations of IA

- Requires extensive CSI
- Can only be achieved for limited number of users K [Yetis,Jafar,Kayran GC'09]
  - For symmetric systems with K users, IA is achievable a.s. iff
    $$N_{\mathrm{T}} + N_{\mathrm{R}} - (K+1)d \geq 0.$$
  - Example with $N_{\mathrm{T}} = 4, N_{\mathrm{R}} = 2, d = 1$: solutions for $K \leq 5$.
- IA requires multi-dimensional fading channels:
  - multiple antennas (space)
  - OFDM (frequency)
  - channel extension (time)

Solutions to IA over the MIMO-IC

- Iterative (slow !) scheme based on the minimization of the interference leakage metric [Gomadam,Cadambe,Jafar IT'08]:

$$I_w = \sum_{i=1}^{K} \sum_{j=1, j\neq i}^{K} \left| \mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ij} \mathbf{V}_j \right|_{\mathrm{Frob.}}^2$$

- Closed-form solutions only for certain K or in particular cases ($N_{\mathrm{T}} = N_{\mathrm{R}} = K - 1$, [Tresch,Guillaud,Riegler SSP'09])

Cellular Network Simulation

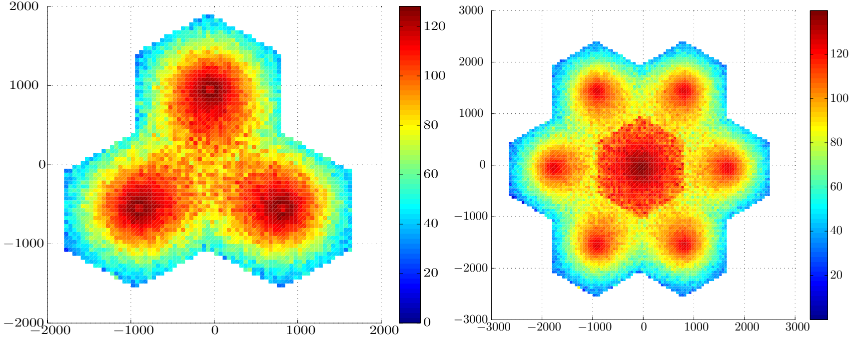
- Compare IA (among K cells out of L) to frequency reuse $1/3$
- Flat-fading channels (no attempt at scheduling)
- Compare the ergodic rate of IA $R_{i,\mathbf{IA}}(p_i)$ to that of frequency reuse + single-link waterfilling $R_{i,\mathbf{FR}}(p_i)$ at each position p_i.

	K	L	N_T	N_R
Scenario A	3	27	2	2
Scenario B	7	37	6	2

Parameter	**Value**
Transmit Power per Subcarrier	$\rho_i = 17\,\text{dBm}\ \forall i$
Path Loss Model	$\gamma_{ij} = 128.1 + 37.6 \cdot \log_{10}(r_{ij}[\mathrm{km}])\,\text{dB}$
Path Loss Exponent	$\alpha = 3.76$
Fast Fading	Rayleigh
Cell Radius	$r = 1$ km
Antenna Radiation Pattern	omnidirectional
Cell Shape	Hexagonal
Frequency Reuse Factor	$\kappa = 1/3$

Cellular Network Simulation Results

$(R_{i,\mathbf{IA}}/R_{i,\mathbf{FR}})$ in % for Scenario A (K=3) and B (K=7).



from [Tresch,Guillaud PIMRC'09]

- Some localized gains (hotspots ?)
- Average gain over space little or negative

IA at *low* SNR: Improve Diversity (play with DMT)

- Let the codimension of the interference-free subspace (at Rx) be greater than the user's transmit DoF: $d'_i \geq d_i$.
  - Letting $\mathbf{U}_i$ $N_{Ri} \times d'_i$ and $\mathbf{V}_i$ $N_{Ti} \times d_i$,

$$\mathrm{IA} \Leftrightarrow \left\{ \begin{array}{c} \mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ij} \mathbf{V}_j = 0_{d'_i \times d_j}, \ \forall j \neq i \\ \mathrm{rank}\left(\mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{V}_i \right) = d_i \end{array} \right\}$$

  - Equivalent channel after interference suppression $\mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{V}_i$ is tall (*diversity*).
- Updated feasibility criterion
  - In the symmetric case $(N_{\mathrm{R}} \times N_{\mathrm{T}},\ d_i = d,\ d'_i = d')$, solution exists a.s. iff

$$d(N_{\mathrm{T}} - d) + d'(N_{\mathrm{R}} - d') - dd'(K-1) \geq 0.$$

  - Not symmetric in $N_{\mathrm{T}}, N_{\mathrm{R}}$: e.g. $d = 1, d' = 2$ is feasible iff $2K \leq N_{\mathrm{T}} + 2N_{\mathrm{R}} - 3$.
- The iterative interference leakage minimization generalizes to this case

IA Maximum Achievable Rate

- With optimum receiver, assuming that interference is not decodable
- Maximum rate achievable by user i:

$$I(\underline{y}_i;\underline{s}_i|\mathbf{H}) = \log\det\left(\mathbf{I}_{d_i} + \mathbf{V}_i^{\mathrm{H}}\mathbf{H}_{ii}^{H}\left(\mathbf{Q}_i^{\mathrm{I}}\right)^{-1}\mathbf{H}_{ii}\mathbf{V}_i\mathbf{Q}_i\right),$$

where

- $\mathbf{Q}_i = \mathrm{E}\left[\underline{s}_i\underline{s}_i^{\mathrm{H}}\right]$ is the covariance at Tx i,
- $\mathbf{Q}_i^{\mathrm{I}} = \sum_{k\neq i}\mathbf{H}_{ik}\mathbf{V}_k\mathbf{Q}_k\mathbf{V}_k^{\mathrm{H}}\mathbf{H}_{ik}^{\mathrm{H}} + \sigma^2\mathbf{I}$ is the covariance of interference+noise at Rx i,
- $\mathbf{H} = \{\mathbf{H}_{ij}\}_{\substack{i=1\dots K\\ j=1\dots K}}$

IA Achievable Rate with Projection Receiver

- Projection receiver

$$\underline{\bar{y}}_i = \mathbf{U}_i^{\mathrm{H}} \underline{y}_i = \mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{V}_i \underline{s}_i + \sum_{j \neq i} \mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ij} \mathbf{V}_j \underline{s}_j + \mathbf{U}_i^{\mathrm{H}} \underline{n}_i$$
$$= \underbrace{\mathbf{U}_i^{\mathrm{H}} \mathbf{H}_{ii} \mathbf{V}_i}_{\bar{\mathbf{H}}_{ii}} \underline{s}_i + \underline{\bar{n}}_i .$$

- Suboptimal but simple
  - interference cancellation in the analog front-end can ease the digital front-end requirements (EC FP7 project MIMAX)
- Ergodic MI is trivially (using truncated unitary $\mathbf{U}_i$ and $\mathbf{V}_i$)

$$\bar{R}_i = \mathbb{E}_{\mathbf{H}} \left[I(\underline{\bar{y}}_i ; \underline{s}_i | \mathbf{H}) \right] = \mathbb{E}_{\bar{\mathbf{H}}_{ii}} \left[\log\det \left(\mathbf{I}_D + \bar{\mathbf{H}}_{ii} \mathbf{Q}_i \bar{\mathbf{H}}_{ii}^H \right) \right] .$$

  - if $\mathbf{H}_{ii}$ is Gaussian i.i.d. fading and channels are independent between users, then $\bar{\mathbf{H}}_{ii}$ is $d_i' \times d_i$ Gaussian i.i.d.

Ergodic Achievable Rates under IA

- Symmetric case with Tx power P spread equally over d signaling dimensions
  - $\mathbf{Q}_i = \frac{P_i}{d_i}\mathbf{I}_{d_i}$, $\mathbf{V}_i$'s truncated unitary matrices (power normalization)
- We want

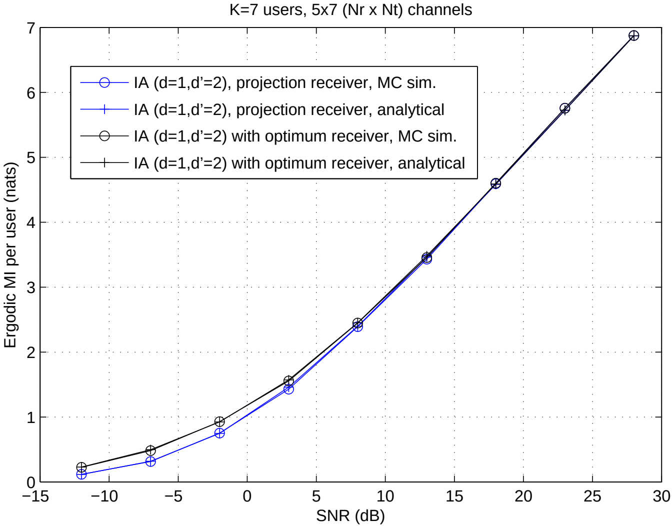
$$R_i = \mathbb{E}_{\mathbf{H}}\left[I(\underline{y}_i;\underline{s}_i|\mathbf{H})\right] = \mathbb{E}_{\mathbf{H}}\left[\log\det\left(\mathbf{I}_D + \mathbf{V}_i^{\mathrm{H}}\mathbf{H}_{ii}^{H}\left(\mathbf{Q}_i^{\mathrm{I}}\right)^{-1}\mathbf{H}_{ii}\mathbf{V}_i\mathbf{Q}_i\right)\right]$$

  - $\mathbf{Q}_i^{\mathrm{I}}$ occupies a random (dependent on the channel realization) subspace of the $N_{\mathrm{R}\,i}$-dimensional receiver subspace.
  - We obtain a tight lower bound

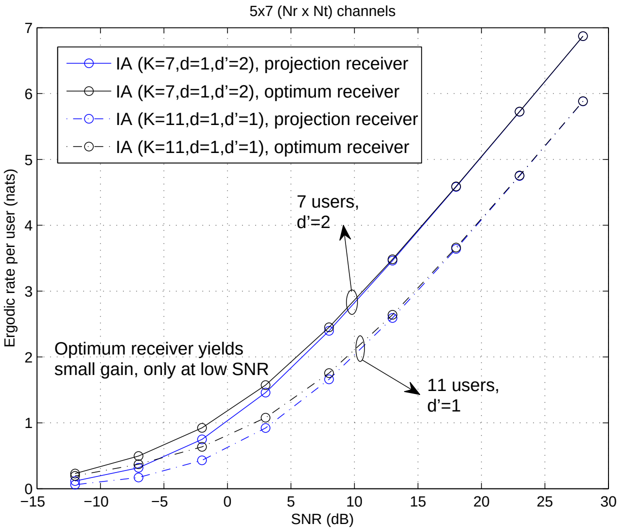
$$\mathrm{E}_{\mathbf{H}}\left[I(\underline{y}_i;\underline{s}_i|\mathbf{H})\right] \gtrsim \mathbb{E}_{\mathbf{H}}\left[\log\det\left(\mathbf{I}_{d_i} + \mathbf{A}\Psi\mathbf{A}^{H}\right)\right]$$

  where $\mathbf{A}$ is a $d_i \times N_{Ri}$ matrix with complex Gaussian i.i.d. coefficients of unit variance, and $\Psi = \mathrm{diag}\left(\frac{P_i}{d_i\sigma^2}\mathbf{I}_{d_i'}, \frac{P_i}{d_i(\sigma^2+\sum_{k\neq i}P_k)}\mathbf{I}_{N_{Ri}-d_i'}\right)$.
  - Evaluated using a recent RMT result [Chiani,Win,Shin IT'10]

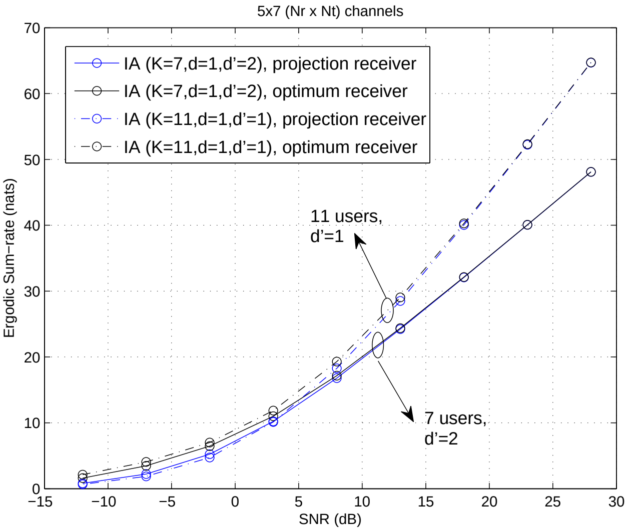
Validation of the Ergodic MI Formulas



Influence of $d' > d$, Per-user Rate



Influence of $d' > d$, Sum-Rate

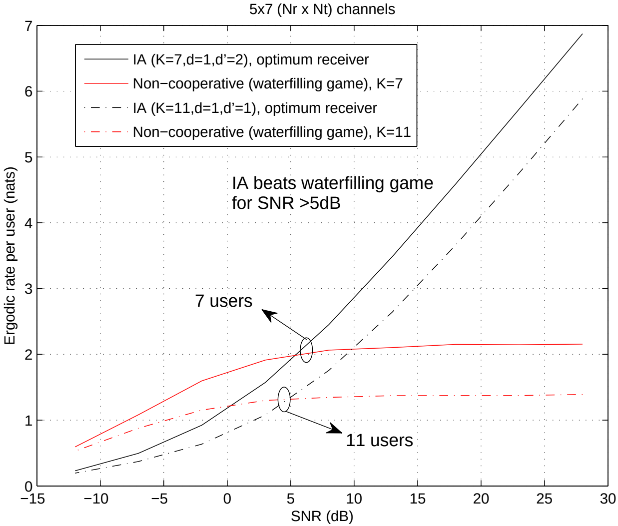


Comparison with Non-Cooperative Methods

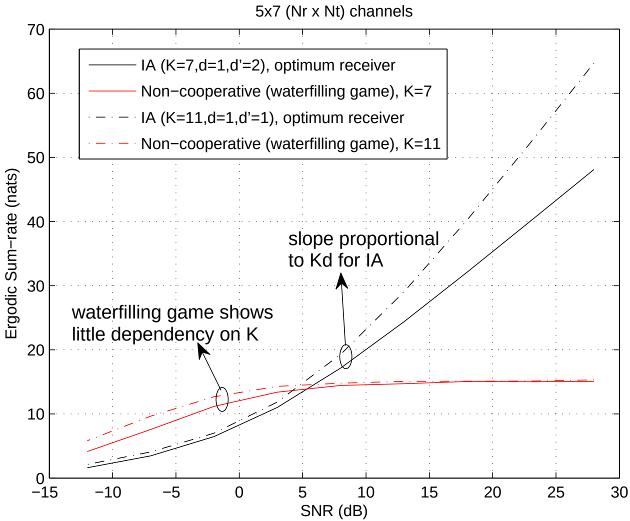
- Comparison with the method of [Scutari,Palomar,Barbarossa IT'09], based on a game-theoretic optimization of the transmit covariances.
  - Each Tx does waterfilling based on the interference covariance at his intended receiver resulting from the previous iterations

- Properties:
  - Inherently distributed method, requires only local channel knowledge
  - Faster convergence than the leakage-based IA algorithm
  - Does not always reach a Nash equilibrium (depending on $\mathbf{H}$). If not, draw another channel realization.

Cooperative (IA) vs. Non-Cooperative (Waterf. Game), Per-User Rate



Cooperative (IA) vs. Non-Cooperative (Waterf. Game), Sum-Rate



Summary

- Analyzed the ergodic performance of IA under Rayleigh fading
  - Closed-form formulas enable asymptotic (in users, antennas) and low-SNR analysis
  - Facilitates optimization of system parameters $(d, d', N_{\mathrm{T}} \ldots)$
- Comparison with distributed transmit covariance optimization (game theory)
- Introduced "diversity receivers" in IA

Outlook:

- Performance of "pure" IA in the low SNR regime is far from spectacular
  - arguably due to the subspace-based definition
  - answer in approximate alignment, properly weigthing the interference power ?

Thank you for your attention

Questions ?

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